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General Editor: JAMES DREVER, M.A., B.Sc., D.Phil.

XVII

FLUCTUATIONS IN HUMAN OUTPUT

BY

S. J. F. PHILPOTT, M.C., D.Sc. (Lond.)

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S. J. F. PHILPOTT, M.C., D.Sc. (Lond.)

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APPROVED FOR THE DEGREE OF
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All tables were re-calculated while the chapters were being set up in print. Cross references have been amended, save in the case of the critical point in the neighbourhood of the 64th second. According to the new figures, it is nearer $63\frac{1}{2}$ seconds, but as it is mentioned so many times, it seemed hardly worth while to amend the text. (In any case, it is not claimed that the values attached to such points are more than approximations. Any alterations have been merely to make them consistent with one another.)

PREFACE

The present research had its inception in an observation made during earlier work on specific fatigue. Watching a subject at work, his waves or pulses of output were seen to slow down as he settled into his stride. The rate of slowing was apparently geometric. In other words, there was here a suggestion that waves in the curve of output are of constant period when plotted against the log of time.

Almost for sentimental reasons, the experiment concerned has been included as constituting Series I of the present research, although it actually took place during the early days of the war. Matters were not definitely put to proof until, the war over, the experiments of Series II and III were performed, methods of analysis elaborated, and a table of log wave-lengths drawn up. (Table II.)

The findings were more important than one had anticipated. Thus, for example, quite a number of the curves seemed to be of one particular log wave-length. Search being made, it was found that curves published by other writers could be said to conform as well. Such an element of invariability was most perplexing. Was it of something in the method of experiment, etc., or had a fundamental law of output begun to emerge? From that moment one has been, as it were, engaged in sorting the pieces of a complex puzzle, arranging and re-arranging them to find a solution.

The second stage of the work began with a search for general and specific elements (Series V). In practice, this became a matter of proving similarities between curves. These were of a somewhat curious nature, and the findings were only explained by assuming (*a*) that all log wave-lengths are whole number multiples of a common unit, and (*b*) that curves of the same wave-length tend also to agree in phase. Even so, however, there were difficulties. To mention but one, there were quite a few curves which, seeming to agree in wave-length, were yet opposed in phase, some having crests where others had troughs, and conversely. (Part II B.)

If the first two stages were of movement, of advance over an open field, the third was of trench warfare, even of rearguard actions. There was yet no proof of geometricity, while the unit of wavelength rested upon curiously precarious grounds. The methods of periodogram analysis and of serial correlation were therefore applied. Save, however, that an unexpected light was thrown upon one or two of the early problems, the results were hardly commensurate with the time and labour involved. (Part II C.)

Finally there was a fourth stage, once more of movement. Returning to earlier conceptions, recalculating both curves and tables, one difficulty after another was resolved. Even cases of apparent opposition in phase were harmonised, and the analysis moved swiftly to its close. Adequately or not, all the pieces of the puzzle of which one had cognizance had been brought into account. (Part I, discussion of Tables III to X.)

It would be almost impossible to render thanks by name to the many people who have helped at one stage or the other by acting as subjects, by helping with the calculations, by criticising. It is gratefully admitted that without such help the work would have been well nigh impossible. Thanks are especially due to the head teachers of Roman Road School, Bow, E. (the late Mr E. Osborn and Miss E. M. Eilbeck), for facilities granted when testing in the boys' and girls' departments respectively, to the teachers concerned, and to the London County Council for permission to do the work there. Thanks are further due to the British Psychological Society for a grant towards the cost of publication.

Within the laboratory at University College I have met with the greatest sympathy and help from all my colleagues, past and present. From Dr J. C. Flugel have come helpful comment and criticism at all stages, while Mr J. T. Raper has been ever ready to take burdens from my shoulders. Finally, the debt to Professor C. Spearman, the head of the department, can never be liquidated.

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Part I

GENERAL HYPOTHESIS

A. INTRODUCTION

The problems here to be discussed are of fluctuations or oscillations in uninterrupted mental work. It is hardly necessary to give a full historical summary in introducing the subject. It has been done so often before. It will be sufficient to indicate briefly the main fields of work, and the nature of the manifestations to be discussed.

No one can "attend" continuously to a task. With the best of intentions, output fluctuates. Such fluctuations can be measured experimentally, and for present purposes it will be convenient to divide the possible experiments into five groups. A subject may be given a simple task to perform, e.g. Addition with one place numbers, output being measured in terms of work done per second, per 10 seconds, and so on, according to the coarseness of the time scale in use. If such measures of output are plotted against time, a "work curve" is obtained, and fluctuations, oscillations, or attention waves are clearly to be seen in it.

A second group of experiments is that of tests in rivalry. If attention is directed to a truncated pyramid drawn in skeleton form, it is seen sometimes as a solid, sometimes as a hollow receptacle. As one alternates from one possible interpretation to another, so attention is said to fluctuate. It should be noted that measures here are of duration only, of times during which the object is seen in its one aspect or in the other. Questions of amounts of work done (of amplitude in the curve of output) do not arise, or rather they are ignored because no measures are possible.

A third group is of tests at the absolute threshold or limen, as with a sound so faint that it can only be heard occasionally. Here, too, measures are usually of duration, of times during which the sound can or cannot be heard. Amplitude measures can, however, be obtained inferentially if desired. The method used by Seashore and Kent (1) was to vary the sound, decreasing or increasing it in loudness, as the subject's ability seemed to rise and fall, the curve of variation in intensity of the sound being taken as a measure of rise or fall in ability.

Fourthly, there is analogous work at the differential threshold. The subject may, for example, be called upon to listen to two sounds of nearby intensities. If the difference between them is small he will sometimes discriminate, sometimes not. Again, measures are of duration, although those of amplitudes could be obtained if desired.

A fifth group of experiments, of work with delayed reaction times, or with judgments of time intervals, will be described in a moment. Here, too, measures are probably of duration.

It is not easy to sum up the findings of previous workers, or to discuss their conceptions of the manifestations. The field is so large; cross-currents so numerous. Generally speaking, however, the problems discussed, and the notions put forward, seem quite definitely to have varied with the kind of experiment performed.

Concentrating upon work at the limen, one comes to think of fluctuation as a tendency for things to move in and out of awareness. As soon as a stimulus becomes supraliminal, this in and out movement ceases. But does fluctuation cease as well? Is it merely associated with the limen? It is here that we get certain controversies. Thus, for example, Cook (2) attacked Heinrich (3) for saying that fluctuations will not come when pure tones are used as stimuli, although they will with noises. Titchener (4) supported Heinrich. Knight Dunlap (5) contradicted them both, showing that it is possible to get fluctuations with pure tones if noise is absent. He suggested that auditory after-images may on occasion bridge periods of actual inability to hear, thus making fluctuations seem to be non-existent. Other discussions of the kind might be quoted. Guilford (6), summing them up, took the point of view that fluctuation is of the limen, adding that investigators who have failed to get fluctuations must have been working under conditions where the limen was difficult to strike.

With researches on rivalry, the conception of fluctuation as a movement in and out of awareness is extended to include a notion of replacement, or of alternation, one mechanism coming into action as another goes out. Thus, for example, Lange (7), working with ambiguous drawings, noted that perception of a skeleton pyramid as a solid may be associated with the notion of a heavy weight, perception of it as something hollow similarly going with the idea of a receptacle, and so on. He therefore argued that oscillations are of central origin, of alternation from one perception to another, of things psychological rather than of things physical. Hylan (8) says that fatigue causes a cessation of the primary activity, allowing a secondary or comparatively separate one to come in, a fluctuation of attention thus resulting. The more interesting the secondary idea, or the stronger the disturbing sensation, the less the degree of fatigue necessary to cause the change. He thus combined the notion of a central psychological cause with that of possible sensory influences.

The work of Dodge (9), although more general in its implication, should perhaps be mentioned here. He says that fatigue is proportioned by the intensity of the stimulus, and that in a complex of conflicting tendencies, the relatively greater fatigue of any one will result in its elimination in favour of a less fatigued one. The problem is of action and reaction in a living organism, where any of the known cycles, cardiac, respiratory, etc., may occasion a change. He points out that the usual test material is not as simple as it might be. Thus, for example, in the Kraepelin sheets usually used for Addition, there are several possible combinations of figures. This in itself permits of, or occasions, switching. He suggests that subjects should be made to do one homogeneous task continuously to see if the fatigue effects do become greater, admitting, however, that heterogeneous tasks could also be fatiguing by reason of their complexity. (In this respect, reference may be made to a much earlier piece of work by Squire (10) who says: "The simple series gave more occasion for fluctuation than did complex. The ease of the series, and the monotony of the task favour mind wandering in the most conscientious subject. With the complex series, peripheral distractions are not so frequently a cause of fluctuation.") Robinson and Bills (11) took up Dodge's suggestion with regard to homogeneous work. They found that the fatigue decrement actually is greater with homogeneous than with heterogeneous work*.

Working with supraliminal tasks, e.g. Addition, Tapping, or the like, one tends to regard fluctuation as a matter of up and down movement in ability rather than of in and out movements in awareness. This is because measures are no longer of durations alone, but of amounts of work done as well. Such up and down movements of ability are clearly seen in work curves, provided the time intervals are small enough. But they occur in curves that convey more than do those of liminal or rivalry experiments, curves showing the slower up and down movements of practice and fatigue. In the literature therefore, we find that fluctuations here are sometimes ignored, or regarded as accidental. In any case, since experimenters have so often returned the figures in terms of intervals of 5, 10 or even more minutes, the waves may be distorted, and not easy to compare with those obtained when intervals of seconds are the rule.

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* The attribution of meaning to one aspect of an ambiguous drawing lengthens the time it can be kept in consciousness. There would here seem to be a possible case of increase in heterogeneity. Cf. work of Lange (7) mentioned above.

The curve of output, as such, is resolved by the Kraepelin school into four or five components. Those of Fatigue and Associative Practice are roughly complementary, the one falling off, the other rising, equilibrium setting in at about the 30th minute or thereabouts. Under the heading of the Apperception curve, come several manifestations. At the outset, there is a stage of rapidly growing, but quickly passing tension, resulting in an Initial Spurt. Following this may be occasional Attention Waves, spurts in which the subject apperceives the task afresh or more clearly than usual. While at the end, there may be an End Spurt where, anticipating the end of the task, he settles down with renewed ardour for a run out to the finish. The Anregungs curve is next postulated, one of long slow oscillations. While, finally, there is the curve of Apperceptive Practice. As the subject works into his stride, he reaches a level at which warmth of familiarity is complete. Beyond that point, the curve in question is assumed to remain more or less at a constant level.

Thorndike (12) pours scorn on the Kraepelin analysis. He says "the most important fact about the curve of efficiency is that it is, when freed from daily eccentricities, so near a straight line, and so near a horizontal line."

Chapman (13) obtained a curve of 10 minutes length, time intervals being of 2 minutes each. The fall in output from interval to interval was surprisingly regular, his curve being essentially smooth, and in outline strongly resembling that for rate of change in a chemical reaction.

Phillips (14) confirmed the work of Chapman, obtaining a curve of 20 minutes length. For the first 14 minutes it is as smooth as that of Chapman. After that, it becomes a little irregular. He actually used 1-minute intervals, values being pooled per 2 minutes for purposes of the above comparison. Plotted per minute, the curve is oscillatory throughout. It drops into a deep trough at the second minute point, rising at once to a marked crest at the third, falling slowly thereafter to equilibrium. Phillips was not able to show that the crest at the third minute is significant, although it is nearly so. He used it, however, in putting forward a theory of the curve, postulating a curve of fatigue and recovery, both hyperbolic in shape, but one upside down to the other. If a slight lag is assumed, one curve hanging behind a little, then on summation, a crest-trough unit of the kind met with might well be obtained. Flugel (15), on the other hand, reports that he could not find this initial manifestation.

Problems of duration or periodicity have usually been associated with those of postulated causes. It has been suggested by Exner (16), Bonser (17), Slaughter (18) and others, that the waves run parallel with those of the Traube-Hering cycle (period 6 to 15 seconds). Taylor (19) and Galloway (20), claim further that if the attention wave lengthens, the Traube-Hering cycle does so as well. Other workers, e.g. Lehmann (21) or Munsterberg (22), relate fluctuations to the respiratory cycle, the last-named writer claiming that he could vary the waves by varying the breathing rate. This would bring in waves of up to 2 or 3 seconds period.

The curves of both Slaughter and Taylor show signs of a longer wave of unknown origin, its period being of about 25 seconds. Slaughter also mentions one of from 60 to 80 seconds, while Seashore and Kent (1) cover the whole range by suggesting periods of the order of seconds, minutes, and hours.

Most of the work on periodicity has been done with experiments on rivalry or the limen. Voss (23), however, used a supraliminal task (Addition), time intervals a fifth of a second. He suggested that the most frequent fluctuations are of $\frac{13}{5} = 2{\cdot}60$ seconds period. Others mentioned by him are of $\frac{3}{5}$, $\frac{7}{5}$ and $\frac{10}{5}$ respectively. The first of these other cycles he explained by saying that it is the average time of doing a single addition. That of $\frac{7}{5}$ seconds, he related to the 1·25 seconds period of Glass (24). With his other waves, he approached those of 2–3 seconds mentioned by other writers.

Thorndike (12) strongly criticised Voss for his method, suggesting that he would be bound to get a most frequently occurring interval even in chance determined material. (He had measured distances between maximal points in the curve.)

The work of Glass mentioned above was on judgment of time intervals. He found that some intervals tend to be assessed more accurately than others, points of maximal accuracy being at 1·50, 2·50, 3·75, 5·00, 6·25, ... seconds. There is here a periodic swinging, the times differing by about 1·25 seconds. Similar work was done by Estel (25) and Mehner (26). Both found a rhythm of this kind, the one saying that maximal accuracy comes with times that are multiples of the Indifference Point value (see discussion below), the other laying stress on odd-numbered multiples thereof, Mehner's figures, for example, running 0·71, 2·15, 3·55, 5·00, 6·40, 7·80, 9·30, ... seconds.

The Indifference Point is that interval of time which is judged most accurately. Quoting from James (27), determinations have been obtained as follows:

Wundt	0·72 second	Stevens	0·71 second
Kellert	0·75	Mach	0·35
Estel (probably)	0·75	Buccola	0·40
Mehner	0·71		

James remarks: "The odd thing about these figures is the recurrence they show in so many men of about three-fourths of a second." Even the bottom figures are, as he points out, submultiples of the apparent standard. A kindred line of work is that in which fluctuations are measured in terms of goodness of reaction time, the stimulus being given at varying intervals after a preliminary signal. If the subject has to wait for an indefinite period, he will sometimes be caught in moments of forgetfulness, i.e. at times presumably representative of troughs in his curve of attention. His reaction times will suffer accordingly. Breitweiser (28) found that the best results come when the interval is of 2–3 seconds.

Returning to liminal work, note may be made of apparent changes in period. Marbe (29) showed that increase in the difference between ring and ground in the Masson disc is associated with an increase in the time of visibility as compared with that of invisibility. Taylor (19) used the ratio "period of invisibility/period of visibility" as a measure of ability. Wiersma (30) found that the mean time of invisibility increases with the difficulty of the task, although it is not merely due to a corresponding loss in the mean time of visibility. The mean total period does not remain constant. It increases as well*. He therefore suggested that the mean total period is independent of the nature of the task, being determined by intensity alone. Pillsbury (31) noted that the ratio "visibility/invisibility" varies from time to time within one and the same sitting at a task, or from one part of the day to another. Since the change is progressive, he suggested that it measures fatigue†. The conception is of course on all fours with those of Taylor and Wiersma, who relate the manifestation to ability. On the other hand, Pillsbury differs in that he considers the total period to be determined by the Traube-Hering cycle, fatigue being seen in what happens within given complete periods.

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* Mean period of visibility = average of the times during which the object was visible, and analogously for mean period of invisibility. The mean total period is the sum of these two means.

† See, for example, the increase in length of rectangles below the line in Fig. 1*B* as compared with corresponding ones above the line. These conventionally represent periods or spells of invisibility and visibility respectively, and the ratio visibility/invisibility as shown there is obviously decreasing with time.

Slaughter (18) described manifestations of a somewhat different kind. Working with the Masson disc, he showed that periods of visibility and invisibility may vary rhythmically in length, being sometimes short, sometimes long. They wax and wane with a period of 10–15 or 60–80 seconds. When examined in respect of this particular rhythm*, periods of visibility and invisibility do not vary inversely, but directly, one tending to be small when the other is small, and *vice versa*. The suggestion put forward was that there may be several waves present, giving rise to interference phenomena.

Summing up the evidence in respect of periodicity, it would seem that waves of round about 2–3, 10–15, 25–30 and 60–80 seconds are characteristic of the work reported.

If there actually are waves of this kind in attention or output, they will be characterised by phase as well as by wave-length. If they correlate with, say, the Traube-Hering cycle, one would infer a random distribution of crests and troughs. Measures of phase will seem to vary from sitting to sitting, or from person to person. From work on time estimation or on reaction times with indeterminate preliminary periods, it would, however, seem that phase must be constant. If, for example, goodness of response in a delayed reaction depends upon escaping a trough in the attention wave, and if there is constancy in the finding that 2 to 3 seconds is a good time to allow, it follows that there must be a corresponding tendency for a crest to appear in the attention wave at about that time. There is constancy of phase within the given limits. A similar argument applies in the case of time judgments.

Turning to problems of amplitude, it has often been reported that waves at the end of a curve tend to move more freely than those near the beginning. Output becomes more variable. Smith (32) used variability in work with McDougall's Dotting machine as a measure of tiredness. Fox (33), using figures of his own as well as those of other workers, showed that much larger coefficients of variability are obtained from the end of a curve than from the beginning. He took the figures for Reed's 10-hour experiment (34), showing that, although mean output for a given period at the end of the day varied little from that of a corresponding period near the beginning of the day, the coefficient of variability had gone up. Flugel (15) noted that the increase may set in from the first few minutes of the curve, and he too attributed it to fatigue.

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* Our Fig. 9*A* is possibly of this type. Round about the 10th second, and again near the 40th, both positive and negative rectangles tend to be small. In between, they tend to be large. The "period" within which waxing and waning takes place is of about 30 seconds.

Another problem of interest here is that of the general factor of oscillation. Using measures of amplitude, Flugel has shown that people who oscillate widely in one thing tend to oscillate widely in all, and *vice versa*. Applying the criterion of tetrad differences, to Flugel's figures, Spearman (35) claimed that there is a general factor of oscillation, symbolised by him as "*o*," a subject's measure of amplitude in any given performance being determined by his "*o*," as modified by his "*s*" or specific factor for that task, the latter depending upon such things as sensory acuity, age, sex, education, and so on.

Finally, turning to our own work, it will be claimed that output curves are essentially periodic (or nearly enough so for them to be treated as such), the waves being geometric in nature, i.e. of constant periods when plotted against the log of time.

Other writers, as for example Gemelli and Galli (36), have shown that pulses may seem to increase in length. The problem in our own work has been to prove the rate of increase to be geometric. Since geometricity is seen in the period or durations of the waves, we have to prove two things together, geometricity and periodicity. Unfortunately, standard methods of proof, e.g. periodogram analysis, call for curves of at least twenty periods length. And that for simple cases. With work curves more might be necessary owing to their complexity. As it happens, the very fact that they are geometric prevents us from ever hoping to get twenty complete cycles of the size usually met with.

The general plan of the work is therefore to present what evidence there may be of periodicity, of wave cycles in the curves. Inter-relationships between the various wave measures will then be noted. Our general hypothesis depends upon these inter-relationships, rather than upon the single cases. And although our primary object was to prove geometricity, it will be seen that the final results indicate a much more fundamental finding, namely the existence of a unit of logarithmic wave-length that is apparently the same for all our subjects under all the conditions of experiment met with.

B. EXPERIMENTAL WORK

1. Experimental series.
2. Methods of handling the figures.
    (*a*) Spell curve method.
    (*b*) Geometrically smoothed work curves.
3. Results.
    (*a*) Arrangement.
    (*b*) Two examples in detail.
    (*c*) Note on remaining examples.
4. Conclusions.

1. THE EXPERIMENTAL SERIES

Five series of experiments were performed. Tasks were of standard type, and a general description will cover them, although there were occasional variations from series to series.

Dotting was on squared paper. The instructions were to begin at the edge of the first line and work across, putting a dot in each successive square. At the end of 5 seconds a signal was given, at which the subjects immediately began afresh at the beginning of the next line of squares, and so on. The mesh or size of the squares varied. For Series II it was 2 mm. With the first Dotting of Series III, it was of $\frac{1}{10}$ inch. In the second Dotting of that series, four kinds of paper were used, meshes being of 2 mm., $\frac{1}{10}$, $\frac{1}{8}$, and $\frac{1}{4}$ inch. respectively.

Arithmetical tasks were of Addition, Subtraction, Multiplication and Division (the last three in Series IV and V only). In Series II, typewritten Kraepelin sheets were used. Subjects worked down each column of figures, adding them by pairs, and entering the answers at the side. Time signals were given at the end of each 5 seconds, subjects thereupon making a tick under the last figure added. For Series III, narrow strips of paper were used, figures being given in rows. Subjects worked across the rows, starting a fresh line with each time signal (always of 5 seconds). This procedure was standardised for later series, printed ribbons of figures being used (ten figures to a line). A further simplification with later series was that subjects merely added two to each number on the ribbon. The other arithmetical tasks were performed in analogous fashion, subjects subtracting two from each number, etc.

Experiments in Discrimination of Sound were performed with a sound hammer in circuit with a fixed resistance and a metronome. By switching the resistance in and out of circuit, noises of two intensities could be made. The test consisted of a long series of noises of the two intensities (in random order), subjects being required to say whether each, as it came, was louder, softer, or the same as the one next preceding it. Time intervals were of 1 second (Series I) or 2 seconds (Series III).

Results from an experiment with the Illusion of Reversible Perspective will also be shown. Times were taken to the nearest half second.

In Dotting and Arithmetical tasks, subjects were asked to work at full speed. No special inducements were offered. Just enough practice was given to ensure that the instructions were understood. One of the special problems in mind at the outset was that of practice effects, and it was desired to get results as low down on the practice curve as possible. From the findings of Series II it became apparent that the major cycles are unaffected by practice. The original procedure was, however, adopted for the remaining series.

It was noted above that the subjects were allowed to begin a fresh line of work with each time signal in some of the tests. This again was because the curves of Series II seemed to be unaffected by the method used. (For Dotting the subjects had begun a fresh line; for Addition they had put ticks after the last figure added, going on from that point with the same line of work.) It is true that by the fresh-line method subjects have the advantage of a visual representation of their output rates before them, the length of successive lines of answers telling them how they are progressing. But with intervals as short as 5 seconds this element of help hardly matters. If a subject is averaging four units of work per time interval his own span of apprehension is wide enough to keep him aware of the fact.

The use of time signals at all is, of course, dangerous. Subjects may develop a rhythm of the interval, or multiples thereof. On the other hand, as long as the work itself is discrete, there will be danger of rhythm. The subject is as much in danger of swinging through his dots or sums in groups of one, two, three or more at a time, as he is of swinging through time intervals in such groups.

No "Flying Starts" were allowed. These have been given by some investigators on the ground that the subjects may have seen the first few units of work before beginning or because it may take them a few seconds to settle down. It will be seen from the present results that much may happen during the first few seconds, and it has seemed more logical to include all the work done. The series of experiments were as follows:

Series I. A few preliminary tests carried out in the Psychological Laboratory of University College, London, 1914–15. Geometricity was suspected, and ratios 1·50 and 1·55 noted. Work at this stage ceased until 1919.

Series II. Two subjects (working in an Active Service Army School, B.E.F., France), sat for an hour, morning, afternoon and evening, over a period of a week. Tasks, Addition and Dotting. Records were of 5 minutes' length, save that each subject did one of 25 minutes in each task. These longer curves are presented with those of Series IV, so that full advantage may be taken of their length. Problems in view were of individual and specific differences, of practice, time of day, etc. Although not part of the research series, a record in Illusion of Reversible Perspective, obtained incidentally from Sm., will be shown. Curves of this series are presented as of Sm. or Ga. respectively. (Jan. 1919.)

Series III. Carried out in the Psychological Laboratory of University College, London, 1919–20. Subjects four boys, aged about 12 years, reinforced for the last test by two others. Tests of Dotting, Addition, Sound, and again Dotting, this last with four kinds of squared paper, to see if size of mesh made any difference. Tests lasted from 6 to 10 minutes. Curves are designated as of Four Boys, or Six Boys.

Series IV. A small collection of longer curves (varying in length from 22 to 60 minutes). They were as follows:

Sm. and Ga. Dotting and Addition (cf. Series II).

Southwark. Addition. Six adults, one record each at the task, working in the L.C.C. Evening Literary Institute, Southwark.

Stepney. Subtraction, Multiplication, Division. Three curves, produced by two girls each (age 15 years) working in the Stepney Day Continuation School.

Pe. Multiplication. A single record from a graduate in Psychology.

U.C.L. Addition. Work done by a group of eight students (one record each), in the Psychological Laboratory, University College, London.

Series V. About 100 boys in the upper classes of Roman Rd. L.C.C. School, Bow, E. Most of the work here was of the sandwiched or alternated variety, and is described in Part II. The curves are labelled Full, Extracted, Straight, and for general purposes may be regarded as ordinary work curves when reference is made to them in the text.

2. METHODS OF HANDLING THE FIGURES

(a) *Spell curve method**.

The general conception is illustrated in Fig. 1. If a subject is made to look at a light of minimal brightness, its intensity being constant, and of a level represented by the horizontal line cutting across curve A, and if his ability varies as shown by the said oscillatory curve *A*, then, as time goes on, he will sometimes see the light, sometimes fail to do so, his successive "spells" of ability and inability beginning and finishing at the points in which the curve cuts the straight line.

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* This method was abandoned later in the research. It was, however, a most useful tool in the early stages.

Spells of this kind are conventionally plotted as in curve *B*, rectangles above and below the line being of positive and negative spells respectively. But such a method fails in many ways to express the facts. Thus, for example, it would appear in this particular case that "trough spells" are increasing in length relative to "crest spells." In a sense, this is true, but it may nevertheless be misleading to say so, since in the original curve *A*, crests and troughs are of equal duration throughout. Curve *C*, in which the rectangles are replaced by equilateral triangles, is better in this respect. The waves are not as distorted, and there is an element of slope in the main curve comparable with that in the original. For practical purposes, squares were used instead of triangles (curve *D*). Summing up the method

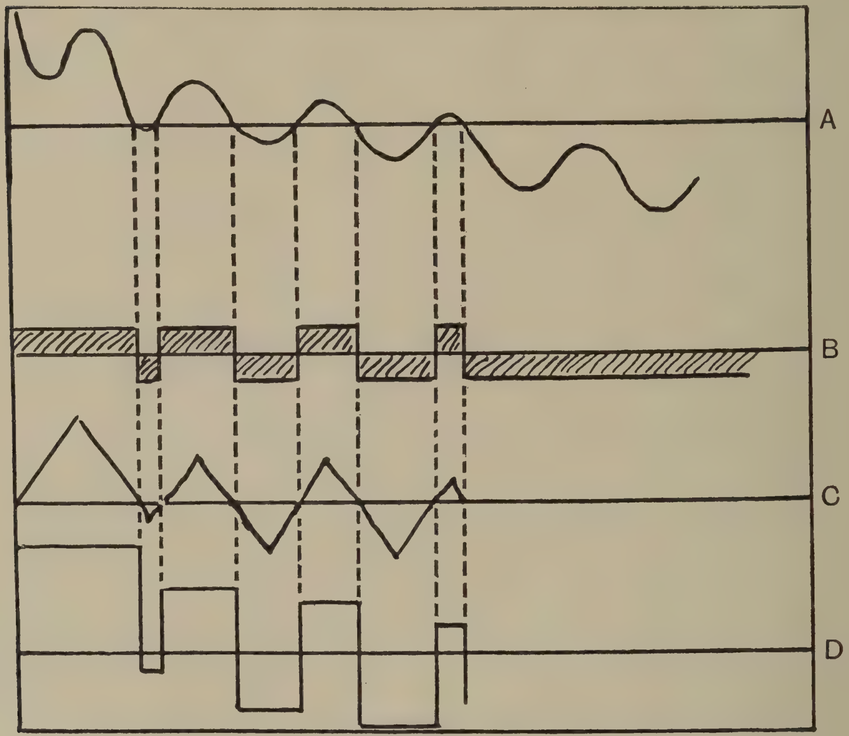

Fig. 1. Methods of expressing a threshold curve.

therefore, it consists merely in replacing the conventional rectangles of curve B by the no less conventional squares of curve D.

The method was employed with work curves as well as with liminal curves. It is true that output figures in a work curve give amplitudes directly. The experimental material is of the curve A type to begin with. Nevertheless, it has been convenient to use the spell method because of the element of smoothing introduced. Work curves are of bold up and down movements as well as of minor fluctuations. Arguing that spells of above and below average work are more likely to be characteristic of the major waves than of minor ones, it seemed that we might emphasise them by returning the curves in terms of spells of good and bad output, thus sweeping away any minor detail*.

If in a test series there are many records, each has to be expressed in this way separately, results then being summed. (It is not sufficient to reduce the total curve to spell form. We need the sum of the several spell curves.) When figures are thus pooled, corners of the squares are rubbed away, and the resulting curve moves as freely as any ordinary work curve, if anything more freely. See, for example, Fig. 2A.

The method has limitations. It will fail to pick out oscillations if the basal curve slopes too steeply. Work curves actually do fall rapidly for the first 30 seconds or so. In that region all values are above the average, and so fused into one long positive spell. But once this first half-minute is over, the curve settles down, and no trouble arises in respect of any small decrement there may be from that point onwards.

(*b*) *Method of geometrically smoothed work curves.*

Fig. 2A has dominant crests centred at about 45, 90, 180 and (possibly) 360 seconds. These times are obviously in geometric progression, with increasing ratio 2·00. Now if such a geometric wave

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* Let the actual output figures in sums or dots per 5 seconds be

$$7\ \ 5\ \ \bar{4}\ \ 6\ \ 5\ \ 6\ \ \overline{4\ \ 3\ \ 4\ \ 3}\ \ 6\ \ 7\ \ 5 \qquad \ldots\ldots(1).$$

The average is five sums or dots per 5 seconds. Overline all figures less than the average. These are of negative spells. The remaining figures are of positive spells. Reducing to the form of squares, the figures may be written as follows (omitting the indeterminate end spell):

$$2\ \ 2\ \ \bar{1}\ \ 3\ \ 3\ \ 3\ \ \overline{4\ \ 4\ \ 4\ \ 4} \qquad \ldots\ldots(2).$$

If these figures are plotted against time, they will plot out the required spell curve.

In one or two cases the spell values (2) were squared, becoming in the above case

$$4\ \ 4\ \ \bar{1}\ \ 9\ \ 9\ \ 9\ \ \overline{16\ \ 16\ \ 16\ \ 16} \qquad \ldots\ldots(3).$$

It was perhaps hardly necessary ever to have done this. On occasion, however, it added strength to the major wave, making it more obvious to the eye. Fig. 2A is of this type.

form occurs in the original work curve (as distinct from the spell curve) it should be possible to make it manifest by smoothing. Such smoothings of the original work curve may be complete (of whole durations) or incomplete (part durations)*. The argument for whole duration smoothing is simple. In a perfect sine wave, the average output over any one complete period or crest-trough unit is equal to that over any other. If therefore there are geometric waves in this present work curve, it should be completely smoothed if we calculate average output rates over complete geometric durations, plotting the

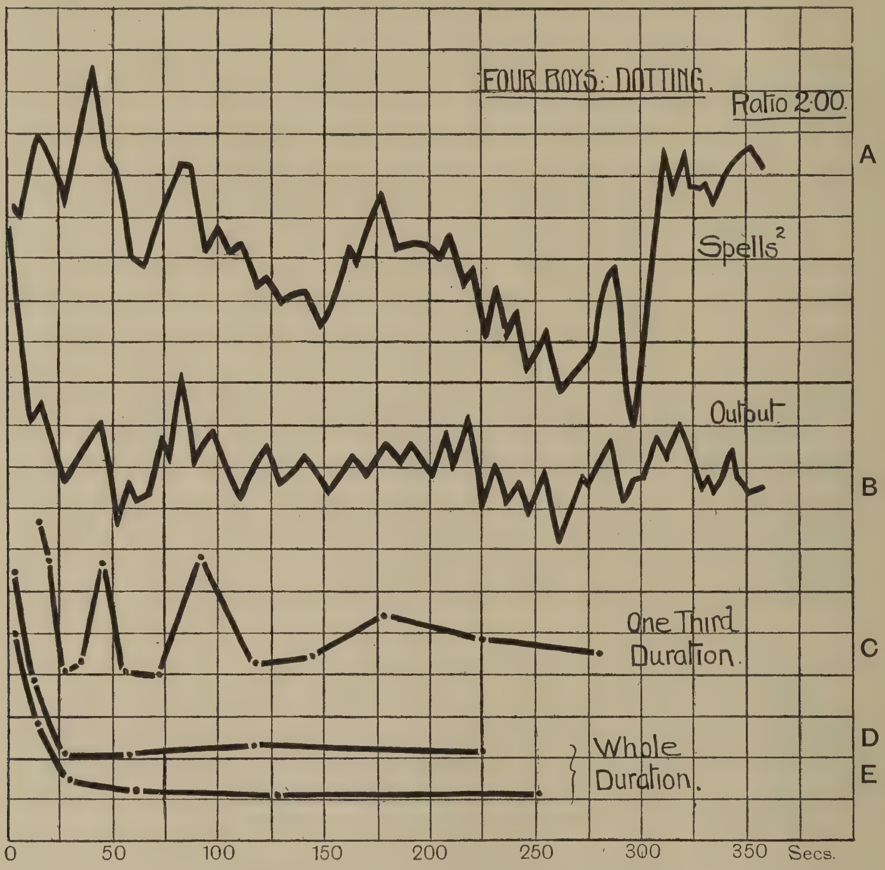

Fig. 2. Example I. Four Boys Dotting.

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* The term duration is used, following Udny Yule. It does not imply the existence of true waves in the mathematical sense. In the circumstances, it is preferable to the analogous term "period."

results against the (geometric) mid-points of the intervals concerned. Treating the work curve (Fig. 2*B*) in this way, curve *E* results. To the eye, at any rate, it is free from oscillations.

The very ease with which smoothness has been attained may detract from its significance, may make it seem that curves ought to smooth in this way. Curve *D* was obtained by shifting the intervals

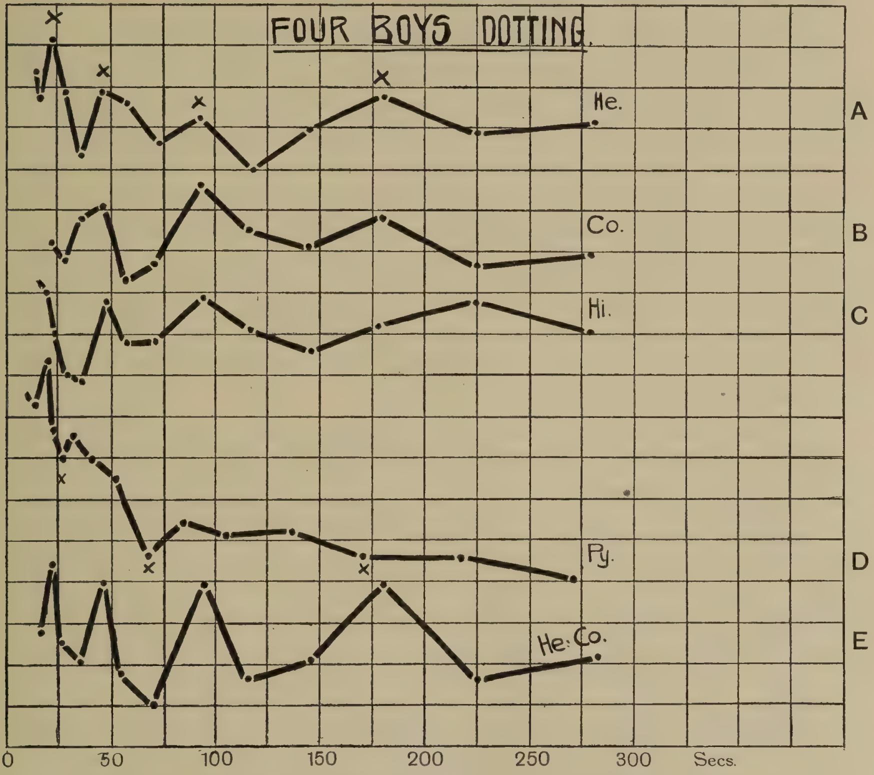

Fig. 3. Example I. Four Boys Dotting.

Work of the four boys individually, intervals as for Fig. 2 *C*, save that those of curve *D* are slightly displaced.

slightly as compared with those of curve *E* (still, however, retaining 2·00 as the ratio). It is far from smooth. See also corresponding curves of Figs. 5, 7 and 8 in this respect.

Smoothness will come with an ordinary sine curve when complete periods are used, no matter how the intervals are placed. It would seem, therefore, that ratio 2·00 does not completely express the present curve. Smoothness does not come with all the possible sequences of intervals. There is, however, one group of cases in which this kind of thing might happen, namely that in which the intervals are of a half duration, one and a half durations, etc. If, taking half-integer values of this kind, intervals are made to begin exactly from the centre of a crest (or trough), smoothness will follow, but only if the intervals are so placed. Arguing thus, ratio $2{\cdot}00^2$, or some other higher power of 2·00, might give better fitting results in this present case, a conclusion to which we shall come again in a moment on other grounds.

Having arrived at complete smoothness, the next step has been to get consistent part duration smoothings. In obtaining Fig. 2*C*, output averages were calculated for three times as many intervals as for curve *E*. That is to say, intervals of ratio $2{\cdot}00^{\frac{1}{3}}$ were employed. Points were so placed that every third one comes squarely in the centre of a presumed crest. The curve as so smoothed should obviously loop by groups of three points each (one of crest height, followed by two depressed or trough values in each complete duration). This is actually so. The outline of curve *C* is just a little irregular, but from the end as far back as the first half-minute, the expected loops of threes actually appear.

Within the first half-minute, the sequence breaks down. The method cannot, however, be trusted as near the origin. If the waves are geometric, there must be an infinite number of complete durations within the first 5 seconds. Even the second 5-second interval will cover at least a complete period of most wave-lengths. Only with the third interval will waves begin to break free. Generally speaking, therefore, anything within the first half-minute or so is suspect. The curve is flattened and distorted by the method of handling the figures.

A problem of a similar kind is that of interaction between smoothing intervals and the waves. If a wave of 4 units period is smoothed in terms of a 5-unit interval, a complex period of 20 units length will be set up, and in practice we might well take such a cycle to be a naturally occurring one. Interference effects of this kind will not be serious with geometric waves once the first few time intervals are past. For as soon as the duration of a crest or a trough is large compared with that of the smoothing interval, the curve merely suffers distortion, the apparent period remaining unchanged. The rough rule again is that the first half-minute covers the danger zone. For these reasons, therefore, we have usually ignored that part of the curve—sometimes more—when calculating geometric smoothings. (In such cases, however, the original work figures (per 5 seconds) have been plotted in order to complete the outline of the curve.)

Rhythmical looping ought not to be expected in every case. Wave systems in work curves are complex, and simple loopings will only come when the given wave is strong enough to dominate all others. In practice, too, the size of the interval has to be considered. If it is too small, minor detail will begin to appear, interfering with the smoothing. One-half*, one-third or one-quarter durations can usually be expected to give fairly good results, provided the basal ratio is not too small. Analogous considerations apply in the case of whole duration smoothings. No one ratio completely expresses an ordinary work curve.

3. RESULTS

(*a*) *Arrangement.*

Having already described one example in brief, the results as a whole may be summed up by saying that they consist of a series of

Table I

Series	Ratios				
	3·02	2·70 2·40 2·52	2·09 2·00 1·94	1·70	1·50
I	—	—	—	—	Ll.W.J. Sound
II	Sm. Dotting	—	Sm. Addition	Ga. Addition	Ga. Dotting
III	Four Boys Addition	Four Boys Sound	Four Boys Dotting (1)	Six Boys Dotting (2)	—
IV	—	Total curve	Sm. { Dotting Ga. { Addition Southwark, Addition	—	—
V	—	?	?	—	?
Supplementary	—	—	Sm. Illusion. Sm. Dotting. Thorndike examples	—	—

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* The terms half-duration, etc., are used, although, strictly speaking, we are dealing with corresponding roots of the ratios. It is more convenient to speak in this way. In any case, we shall frequently be using logarithmic time intervals, when the terms have their usual meaning, and can legitimately be used.

such examples (Table I), most of them set out as in Fig. 2, and each manifesting some one or other of the major geometric waves met with. Our primary object in this section is the presentation of evidence for wave systems as such, but the curves are given in a form such that light is thrown on other problems as well. Curves of Series II are given per subject, possible individual differences being in mind. Those of Series III are per task (one twice repeated) to illustrate possible specific differences. Only two examples are dealt with fully in the text. The others are then briefly mentioned, further details being given as occasion arises.

(*b*) *Two examples in detail.*

Example 1. Four Boys Dotting. 19 records. Figs. 2, 3, 4.

Example 2. Sm. Ga. Addition, Dotting. 4 records. Southwark. Addition. 6 records. Figs. 5 and 6.

The curves of Fig. 2 have already been briefly described. Individual results for the four boys concerned are given in Fig. 3. They are of one-third duration smoothings, intervals as for Fig. 2*C*, save that those of Fig. 3*D* have been shifted slightly. Subjects He. and Co. conform fairly well to the average pattern. Adding their curves together, Fig. 3*E* is obtained, undoubtedly a good curve of its kind, one looping regularly even as far back as the 22nd second.

The curve of subject Py. loops by fours and twos rather than by threes. The ratio is thus $2{\cdot}00^{\frac{4}{3}} = 2{\cdot}52$, or the root thereof $= 1{\cdot}59$. Incidentally it should be noted that troughs are more characteristic of this curve than crests, as they are with curves of all ratios other than ratio 2·00. There is weakness in the trough presumed to occur at the 170th second. The point is of interest. A ratio 2·00 system seems to call for a crest at 180 seconds, one of ratio 2·52 similarly demanding a trough at 170 seconds. If both are present, there will be interference, the zone becoming indeterminate. (Weakness in this zone is characteristic of many curves.)

The curve of subject Hi. is of yet another type. From the 45th to just short of the 180th second, it is apparently of ratio 2·00, but outside that zone the threefold system of loops breaks down. The quickest way of dealing with this example is to return to the original work curve. This is given among those of Fig. 4, plotted against the log of time, geometric intervals thus seeming to be of equal duration. As so plotted, the wave is quite clearly of ratio 2·40, or thereabouts, as shown by marks made under the curve.

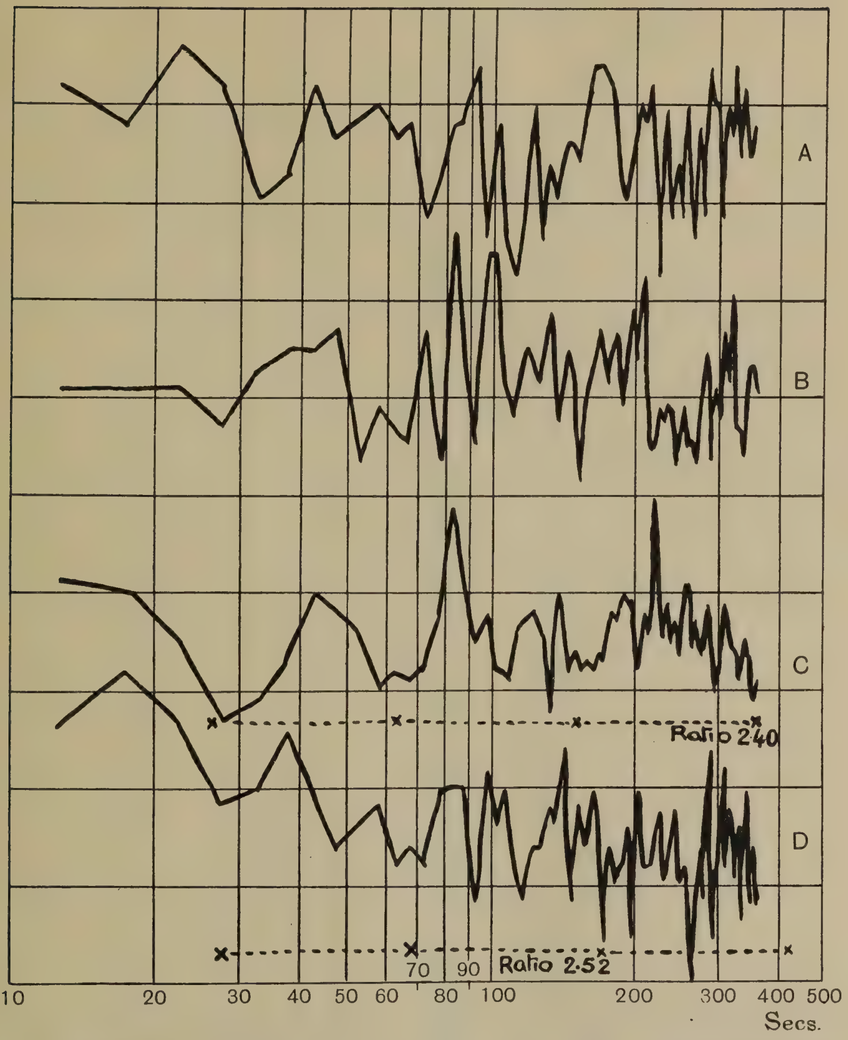

Fig. 4. Example I. Four Boys Dotting. Output curves for the four boys, plotted against the log of time. (Order of the curves as in Fig. 3.)

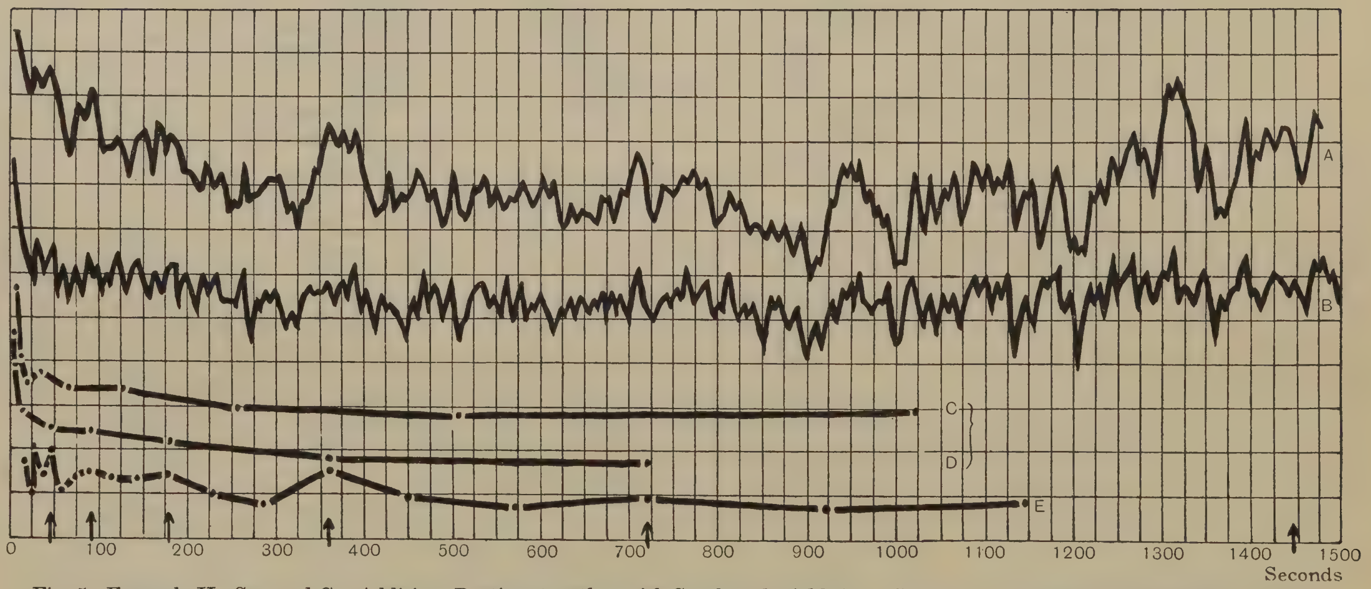

Fig. 5. Example II. Sm. and Ga. Addition, Dotting, together with Southwark, Addition. Curves *A* to *E*: spells, output, whole duration smoothing (two curves) and one-third duration smoothing. With *E*, intervals, etc. are as for Figs. 2 C and 3.

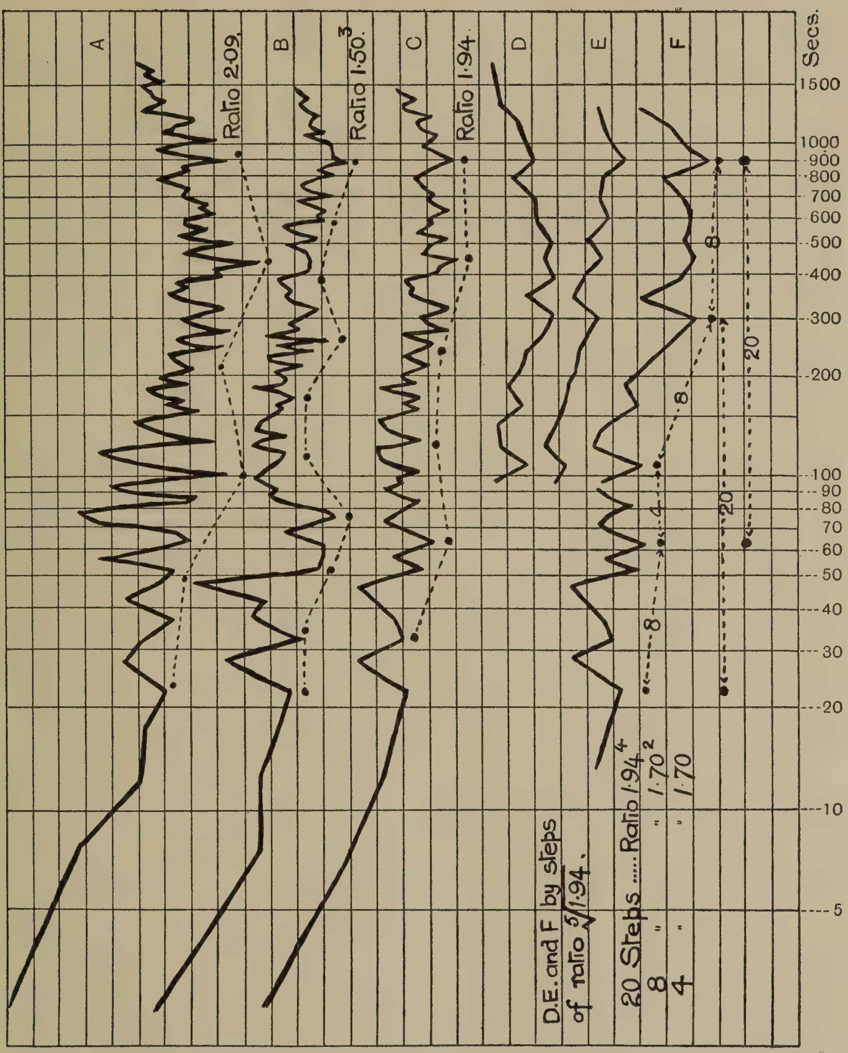

Fig. 6. Example II. Curves *A* and *D*, Southwark, Addition; *B* and *E*, Sm. Ga. Addition, Dotting; *C* and *F*, sums of *A* and *B*, *D* and *E*. Curves are of original 5-second intervals for 100 seconds, thereafter by average output per interval of 0·0160 on the scale log (*A*, *B*, *C*), or per interval of 0·0576 (*D*, *E*, *F*). (Note that 0·0576 is one-fifth the log of ratio 1·94, and one-quarter that of ratio 1·70.)

The diagrams of Fig. 4 exemplify the use of logarithmic time intervals. Drawn in this way, curves are often more easily read than when extended in arithmetic intervals, although the rule is not general. Log work curves can seem to be as erratic as ordinary ones.

Example 2 is of ten records, representing the work of eight adults at two kinds of work. The spell curve (Fig. 5*A*) is of the same type as that of Fig. 2. The first few crests at 45, 90, 180, ... seconds are easily picked up. After that it has to be remembered that we seek spells of long duration rather than outstanding single-point values. When a crest-trough unit extends over 1000 seconds or so, it rises and falls so gently that it may not at first be distinguished from the line of mean level. Bearing that in mind, crests may be traced at about 750 and 1500 seconds, the first-named being somewhat damped, occurring as it does near the lowest point of the curve.

The work curve itself has no very distinctive features. Whole duration smoothing was attained as easily as with Example 1. As with that earlier example, a slight shifting of the intervals upsets the smoothing, ratio 2·00 not completely expressing the curve. Curve *C* is of one-third duration smoothing, intervals being in the same positions as with Fig. 2*C*. The pattern in the curve is somewhat more complex than before. Nevertheless, each of the presumed crest points is definitely higher than trough points on either side of it, the resulting sequence of loops of threes only breaking down as we enter the first 30 seconds of the curve. Here, as with Example 1, there is a trough at $22\frac{1}{2}$ seconds instead of a crest. Apparent irregularities of outline in this curve are chiefly of a tendency to double humping, crests at $27\frac{1}{2}$ and 45, or at 90 and 180 seconds going together. Such manifestations occur also in other curves, see, for example, pairing of the 45–90 seconds crests in Figs. 8*C* and 9*D*. We thus arrive again at the fact that ratio 2·00 does not completely express curves of this family.

Granted that a curve has been expressed in terms of a periodic sequence of points, we may enquire whether such an arrangement could have arisen by chance. From the 30th second onwards, Fig. 3*E* is practically horizontal, consisting of ten points of standard deviation 0·509. Between the lowest crest and highest trough point is a gap of no less than $1{\cdot}495\sigma$, measures being as follows:

Crest zone (above $1{\cdot}24\sigma$ from the mean) normal frequency	10,776
Middle zone	49,276
Trough zone (below $-0{\cdot}255\sigma$ from the mean)	39,948

A fortuitously determined curve of 100,000 points should have

49,276 within a middle zone of this width. Among ten consecutive points we find none, an arrangement that could only occur about once in a thousand times, and that without taking account of the rhythm. If that is brought in, the possibility of a chance origin recedes to about one chance in half a million. With Fig. 5*C* some of the later crests are lower than trough points occurring nearer the origin. Nevertheless, from the 45th to about the 470th second, there are twelve points in which no crest value is absolutely lower than any of the trough values, although it is true there is no appreciable middle zone. Making the simplest assumption, namely that points in such a curve are as likely to be of crests as of troughs, the probability of a threefold looping works out at about one chance in four thousand. The odds are high, and remembering that the material is quite independent of that of Fig. 3*E*, being produced by different subjects (adults instead of children), it is clear that the combined odds are enormously against the rhythm being of chance origin.

Figures of this kind do not prove that a regular sine wave is present. But they do show that the up and down movement is significant. Note may further be made that whole and part duration smoothings are thus shown to be in entirely different categories. The one is smooth to the eye, the other significantly oscillatory. Were this a matter of the degree of smoothing applied to fortuitously arranged ups and downs, the two kinds of curves ought not to have been as unlike as they are. For intervals near the beginning of whole-duration curves are distinctly smaller than those near the middle or ends of part-duration curves. Why therefore is there complete smoothness in the one case, and significant looping in the other? The only simple explanation is in terms of a wave theory, whole-duration smoothings clearing the curve by equating crests with troughs; part-duration smoothings emphasising the waves by setting crests over against troughs.

Fig. 6 is of Example 2 resolved into its two component curves. There is here illustrated the use of small intervals in part-duration smoothings. For curves *A*, *B* and *C* they are of one-nineteenth durations of ratio 2·00. (The reason for this measure will become apparent later. In log units, it is of 0·0160.) To save space, points have been plotted against the log of time, geometric intervals thus being made of equal size throughout. With units as small as these, it is not worth while to begin smoothing too near the origin. The original (5 second) output values were therefore plotted for the first 100 seconds of the curve. At that point, a log unit of the given size is of approximately 5 seconds, and the change over could conveniently be made.

Marks under the curves indicate the most obvious loopings. Curiously enough, they are not of ratio 2·00. With curves *A*, *B* and *C*, they are of 20, 11 and 18 units, corresponding with ratios 2·09, 1·50 and 1·94 respectively. With ratio 2·00, loopings would of course have been of 19 units. Two of the curves conform fairly well, the other seeming, however, to break away completely*.

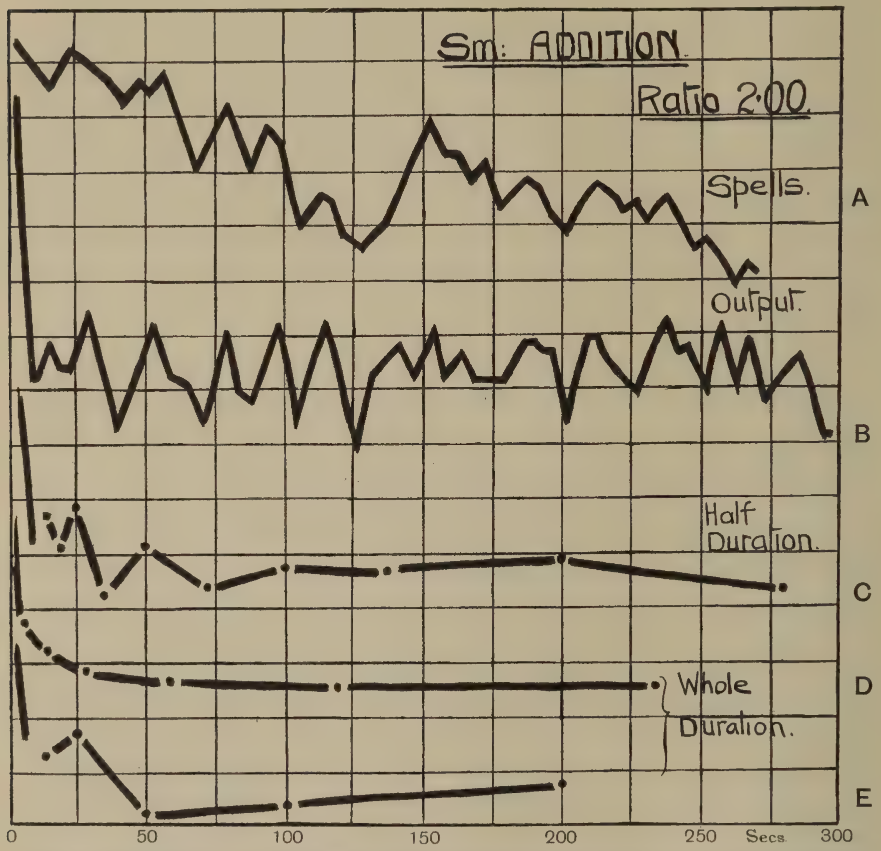

Fig. 7. Sm. Addition. Note that half-duration intervals are used in the *C* curve as compared with one-third durations in Figs. 2 and 3. Later analysis: troughs at approximately 38 and 128 seconds seem to be more of ratio 3·02. See Fig. 11.

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* The break is possibly more apparent than real. See smoothing of this curve with half durations of ratio 1·94, Fig. 31 *B*. It moves quite well by loops of four points each, ratio $1{\cdot}94^2$ being indicated. The case is discussed again later.

There are tendencies to looping in terms of powers of the nominal or basal ratios. Curve *B*, for example, has waves of ratio 1·50 that are themselves grouped by threes, ratio $1{\cdot}50^3$ thus being indicated. That this higher cycle is present seems further to be shown by the fact that the curve will not smooth by whole durations of ratio 1·50. The said curve (not shown here), loops by threes. Smoothness only comes with whole durations of ratio $1{\cdot}50^3$. In similar fashion, curve *A* looks as though it might be looping by double waves. Such a higher cycle would be of ratio $2{\cdot}09^2$. Here, however, whole-duration smoothings of the first-power ratio are indeterminate. There is no definite evidence of a twofold looping.

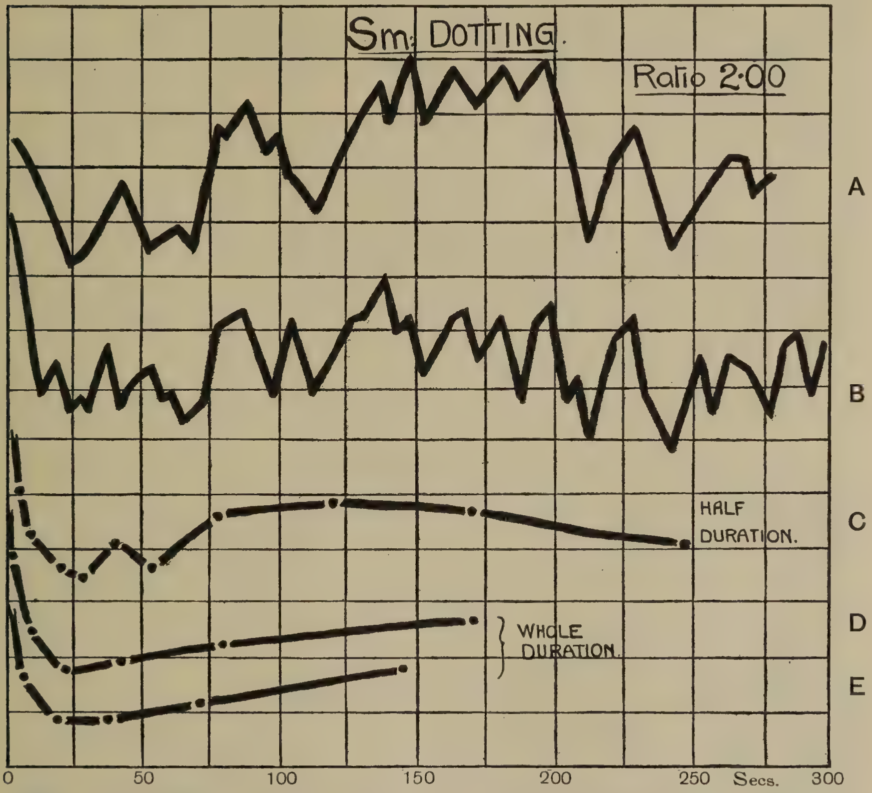

Fig. 8. Sm. Dotting. Curves as for Fig. 7. Note tendency to form double humps. See Fig. 5 *E* for such manifestations between 22½ and 64, or 64 and 270 seconds. (In the present curve, the 22½–64 second pair of crests is best seen in curve *B*.) Later analysis: emphasis is laid on troughs at 64, 215 and 240 seconds approximately (the last two cutting off a characteristic crest). Compare present *B* curve with Figs. 5 *B* and 12 *B*. See also p. 51.

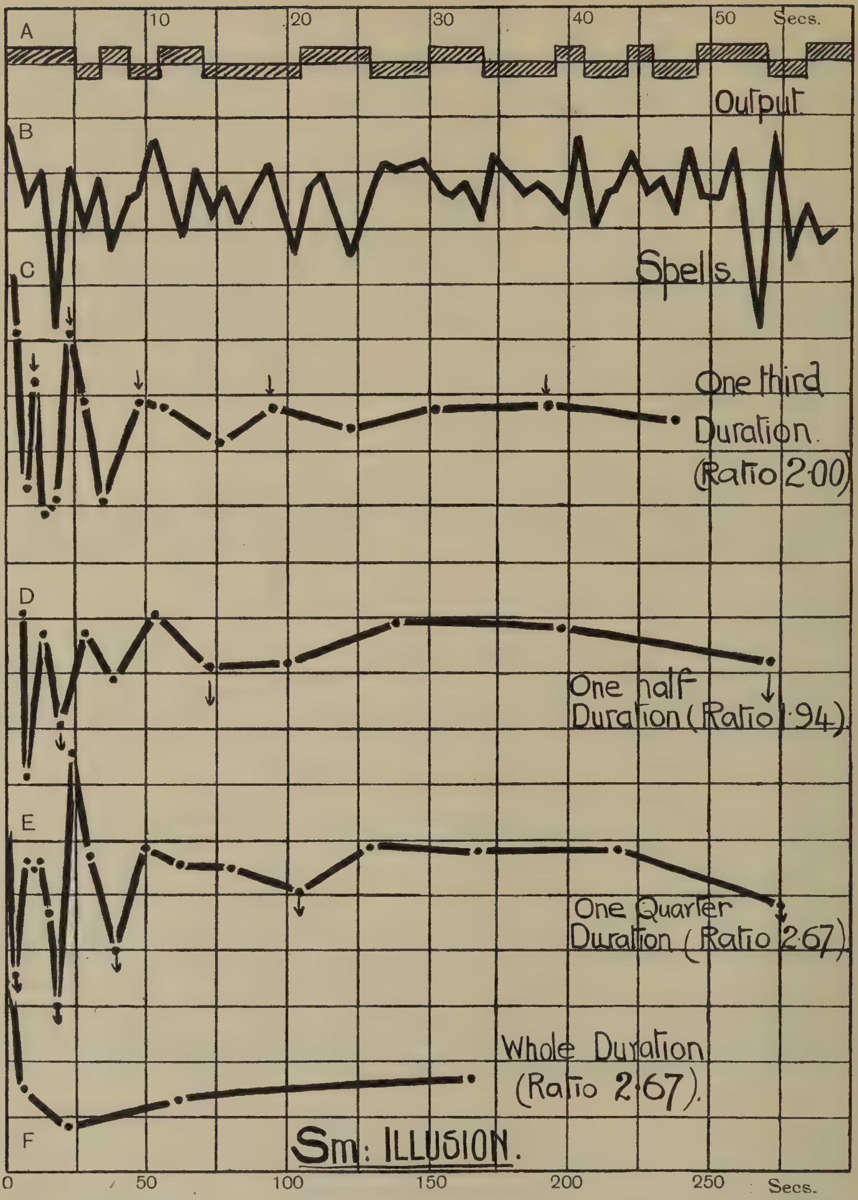

Fig. 9. Sm. Illusion of Reversible Perspective. Note that curve *A* is not on the same scale as others. Compare with Figs. 2, 3 and 5 for one-third duration smoothings of ratio 2·00, and with Fig. 7 in respect of 38–128 seconds zone (in the *B* curves concerned).

Curves *D*, *E* and *F* correspond with *A*, *B* and *C* above them, smoothing intervals now being of one-fifth durations of ratio 1·94. The interval is still rather small to ensure regular loopings, but a strong tendency to twentyfold arrangements can nevertheless be traced, together with minor cycles of eight and four units respectively. From the twentyfold looping, one would infer the presence of ratio 1·70*. A fourfold looping would be of ratio 1·70 itself, an eightfold one of ratio $1{\cdot}70^2$. It is curious to find that signs of those ratios are here stronger than those of ratio 1·94 (which would be seen in fivefold loopings), considering the strength of the manifestations of ratios 2·00 or 1·94 in other expressions of the curves.

With this we may perhaps leave Examples 1 and 2. By successive smoothings we have travelled some distance from the first crude identifications in terms of ratio 2·00†, bringing into account ratios 1·94, 2·09, 1·50, 1·70, 2·40 and 2·52, as well as some of their powers. Table I will show that we have thus illustrated most of the ratios there said to be of importance.

(*c*) *Note on remaining examples.*

The two examples above having enabled us to illustrate most of the major ratios, it is perhaps safe, for present purposes, to present the remaining examples merely as a series of plates (Figs. 7 to 19), relegating any details that may seem to be necessary to Appendix A. Note must be made, however, of certain points arising out of their analysis. On the whole, phase relationships have seemed to be constant. With the ratio 2·00 group for example, crests have always tended to appear at 45, 90, 180, ... seconds, save in a couple of cases where there has been apparent opposition, troughs coming at those points. Similarly with cases of other ratios. There has been a standard trough sequence for each, and with each any deviations from the normal have tended to be of complete opposition‡. It seems obvious that there is here a problem of fundamental importance. Why should there be constancy of phase from curve to curve? There must be some kind of a law behind the manifestations.

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* If two waves interact, they will generate a composite wave of period equal to the L.C.M. of the two periods concerned. Thus waves of four and five units period give a common cycle of twenty units length. The unit of periodicity with the curves under discussion is of "one-fifth durations of ratio 1·94" (see footnote, Fig. 6). But this is the same thing as "one-quarter durations of ratio 1·70." Whence, if ratios 1·94 and 1·70 are present together, their common cycle must be of twenty such units, i.e. corresponding with a geometric wave of ratio $1{\cdot}94^{\frac{20}{5}} = 1{\cdot}70^{\frac{20}{4}} = 1{\cdot}94^4 = 1{\cdot}70^5$.

† All identifications in terms of ratio 2·00 are suspect, although for convenience we have kept to it with short curves.

‡ See, for example, Figs. 10, 18 (*E* and *F*), 11*A* and 12*A*, etc. For mention of other cases, cf. Flugel(15), p. 62.

Problems of an entirely different kind are illustrated by the curves of Figs. 9 and 10. The one example is of Illusion of Reversible Perspective, the other of material borrowed from an article by Thorndike.

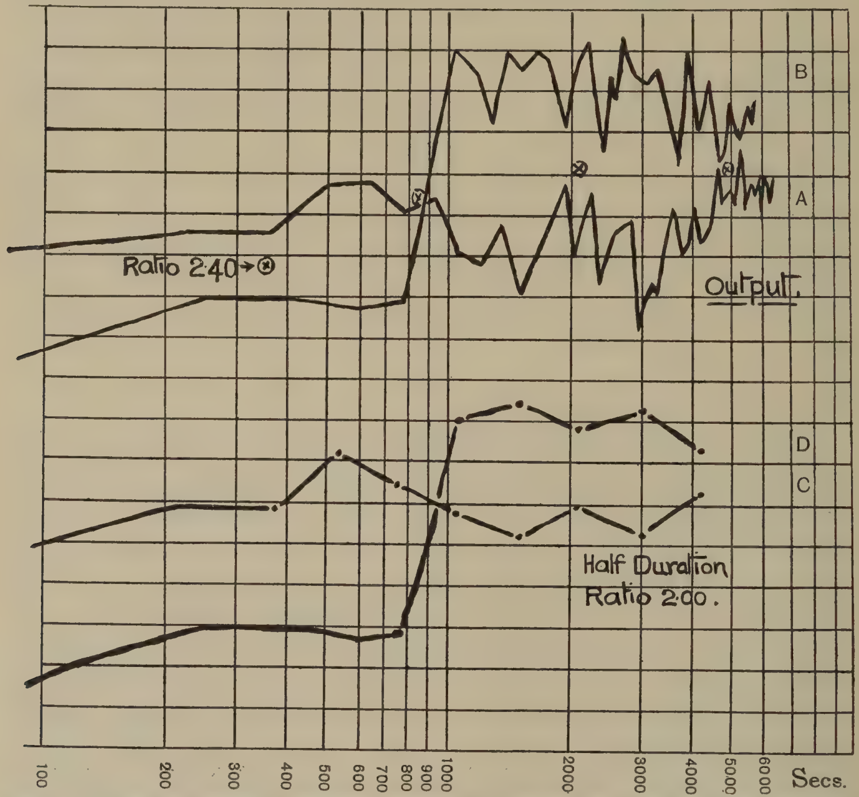

Fig. 10. Thorndike material. Addition. Subject Mc. Curves are of first and third attempts at the task. (See Appendix A.)

In the one case, the subject, as it were, made his own times, no signals being given by the experimenter. In the other, scores were of times taken to do unit amounts of work (times varied between about 80 and 180 seconds). In both cases, conditions were very different from those in our other experiments where scores are based upon work done in unit time (usually 5 seconds). Yet in both Figs. 9 and 10, the cycles conform to standard types. If there are any differences set up by differences of method, etc., they are not manifested in our geometric wave measures.

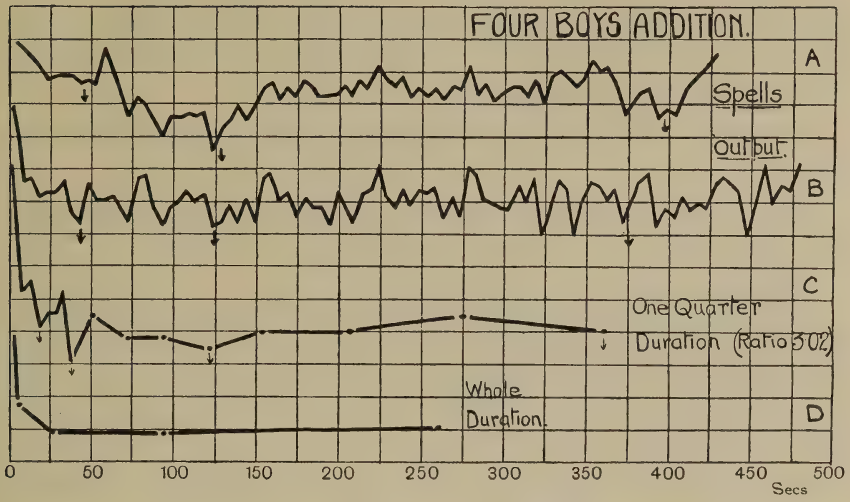

Fig. 11. Four Boys Addition. Standard example of ratio 3·02. Compare *B* curve with Figs. 7 *B* and 9 *B*.

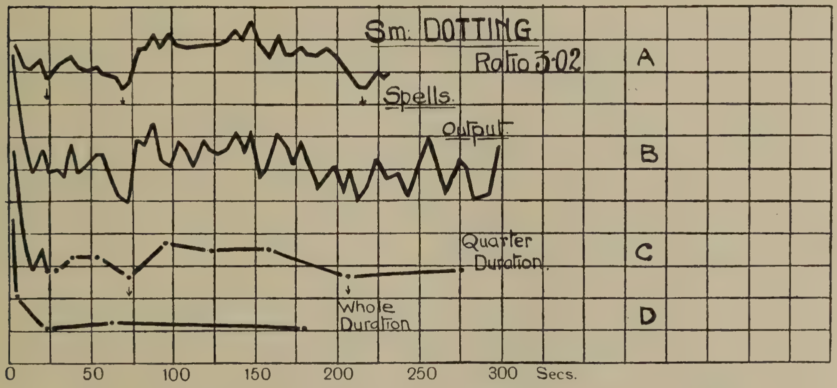

Fig. 12. Sm. Dotting. First analysis (here illustrated): Ratio 3·02, but curves opposed in phase to those of Fig. 11. Later analysis: more of ratios 3·45 and 3·77 See Fig. 8 and also p. 62.

4. CONCLUSIONS

The curves of the research behave as though they contain waves of geometric duration or period. The said waves are of major fluctuations, showing in the presence of fairly wide zones of good or bad output. Evidences are as follows:

(*a*) Spell curves are often, although not invariably, geometric to the eye.

(*b*) When the original work curves are averaged in terms of whole durations of the dominant cycle peculiar to them, they tend to become completely smooth.

(*c*) When similarly averaged in part durations, work curves tend to loop rhythmically, provided always that the interval used is not too small. With small intervals, the loopings suggest the presence of interference patterns, of complex L.C.M. cycles based upon more elementary waves.

If two or more curves are of the same ratio, they tend also to agree in phase, within a few seconds either way. Larger differences sometimes occur, but if so they tend to be of simple amounts, usually of a half duration.

Agreement in wave-length and phase does not imply similarity in outline. In other words (assuming our measures to be of complex L.C.M. cycle lengths), curves may be of the same general type as far as major ratios are concerned, while yet differing in constitution, that is to say, in the combination of elementary waves going to their make up. Such differences in outline (or in constitution) must cover any possible differences due to individual peculiarities from subject to subject, or from task to task, etc., measures of cycle length and phase appearing to be constant and of universal significance—within the limits of experimental error.

C. RELATIONSHIP BETWEEN THE RATIOS

1. Tabulations.
2. Interpretation of the tables.
    - (*a*) Common multiple cycles.
    - (*b*) Cycle frequencies: hypothetical and actual.
    - (*c*) Cycles in strong and weak curves.

3. Apparent phase relationships.
    - (*a*) The problem.
    - (*b*) Inter-cycle relationships: the common trough point T_0.
    - (*c*) Trough sequences of the various ratios.
        - (i) The argument.
        - (ii) Sequences of the ratio groups.
    - (*d*) Common trough zones: hybrid cycles.
    - (*e*) Tentative criticisms.
4. Conclusions.

1. TABULATIONS

When the research had progressed sufficiently to allow of a tabulation, more than half the then determined ratios were found to fall into a series such that their logs advanced by steps of 0·0480 from zero. It is true that some would not conform, but to find more than half do so seemed significant. Later it was noticed that if the interval were reduced to 0·0160, it would be possible to include all but a mere two or three values. The first thirty values of the series so obtained are given as Table II.

An attempt will be made in a moment to interpret the findings, recasting the table in so doing. One or two problems may, however, be mentioned to begin with, problems attaching to the figures as they stand. Although a column of differences is given, no stress can be laid upon it. Ratio 2·00, for example, was obviously taken to the nearest whole number. Or again, some of the omitted ratios were possibly alternative renderings of others brought into the table, the good fit of those included values being to that extent discounted*.

The two ratios about whose fit we were most concerned were 1·30 and 1·70 (1·69 or 1·68). First noted in Fig. 14*A* they were identified just after the 0·0480 series had been noted. They did not fit into it of course, but even when the interval was reduced to 0·0160 (a change really made to bring them into series with the others) they still fitted less easily than most. The problem was a curious one. Here were the first ratios to be determined after the tabulations were begun, and they could not be made to conform, even although one might have been tempted to force them into so convenient a mould. The least they could be made in log measures was 0·2260 and 0·1130. But there was also evidence going to prove a relationship with ratio 1·94 (cf. p. 27), and as this would bring their logs up to 0·2304 and 0·1152, the discrepancy with the table became even greater than at first. Whatever the ratios may be, their logs are not 14 and 7 times 0·0160.

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* From notes made at the time, ratios 1·32, 1·38 and 1·49 seem to have been the only ones left over. There were possibly a few more. The ones mentioned may be alternative readings for ratios 8, 9 and 11 of the table.

Table II

Ratio no.	Ratios	Log *R* (empirical)	Log *R* (hypothetical)	Difference
0			0·0000	—
1	×	×	0·0160	—
2	×	×	0·0320	—
3	1·116	0·0476	0·0480	−0·0004
4	×	×	0·0640	—
5	×	×	0·0800	—
6	1·245	0·0951	0·0960	−0·0009
7	1·30	0·1136	0·1120	+0·0016
8	1·34	0·1271	0·1280	−0·0009
9	1·39	0·1440	0·1440	0·0000
10	1·44	0·1584	0·1600	−0·0016
11	1·50	0·1760	0·1760	0·0000
12	1·55	0·1903	0·1920	−0·0017
13	(1·63)	—	0·2080	—
14	1·69	0·2272	0·2240	−0·0032
15	1·73	0·2380	0·2400	−0·0020
16	×	×	0·2560	—
17	×	×	0·2720	—
18	1·94	0·2880	0·2880	0·0000
19	2·00	0·3010	0·3040	−0·0030
20	(2·09)	—	0·3200	—
21	2·16	0·3345	0·3360	−0·0015
22	(2·25)	—	0·3520	—
23	×	×	0·3680	—
24	2·40	0·3802	0·3840	−0·0038
25	2·52	0·4013	0·4000	+0·0013
26	(2·63)	—	0·4160	—
27	(2·70)	—	0·4320	—
28	×	×	0·4480	—
29	×	×	0·4640	—
30	3·02	0·4800	0·4800	—
etc.	etc.	etc.	etc.	etc.

Missing values ×. Any added since table was drawn up are given in brackets, no empirical logs being quoted.

If there are log wave-lengths that do not fit into the table, it may be that more than one series should be postulated. Such an additional series is suggested by the two anomalous ratios above. If their logs are taken as 0·2304 and 0·1152, a series of log increment 0·0144 can be constructed into which quite a few ratios will fit. Thus, for example, the logs of 1·30, 1·39, 1·49, 1·70 and 1·94 are 8, 10, 12, 16 and 20 times that unit. The conception here is of a second chain of values interlacing with the original 0·0160 series in that 1·39 and 1·94 have logs that are exact multiples of either unit. Other such series are of course possible, although, as it happens, the two mentioned are the only ones covering enough ratios to make them worth while.

Another line of thought arises out of the fact that the first chain to be noticed was of 0·0480. This covered most of the major ratios. But we might have taken the increment to be 0·0960 without losing too many of the more important values. (Most of the major ratios are covered by the 6th, 12th, 18th, ... members of Table II.) Retracing now the steps, reducing the interval from 0·0960 to 0·0480 and finally to 0·0160, a few major ratios are brought in with each reduction, but so few in comparison that with the last step a practical limit is reached beyond which it is hardly worth while to go, empirically at any rate.

Summarising therefore, the first conception was of several possible chains of log wave-lengths, interlacing or intersecting with one another. At the same time, one realised that ratios are not randomly scattered, even within any one chain or series, e.g. that of 0·0160. The more important ones tend to be equally spaced. An explanation is required that shall embody these various notions in one comprehensive scheme.

2. INTERPRETATION OF THE TABLES

(*a*) *Common multiple cycles.*

A possible explanation is as follows. Let it be assumed that a pool of wave-lengths exists with periods evenly enough distributed for the common interval to serve as a unit of wave-length. Assume that for any one work curve, some, although not necessarily all of the possible waves will appear. Let all have an equal chance of appearing, their emergence being chance determined, individual waves not being specific to individuals, to the task performed, or to any recognisable factor of the kind. Finally, keeping matters as simple as possible, let amplitude and phase be assumed to remain constant.

If the assumed wave-lengths are multiples of the common interval, they may be divided by it throughout, thus being reduced to or expressed as natural numbers. If several waves are present, an L.C.M. cycle will be set up. Wave-lengths 3, 4 and 5, for example, will come together, generating a common cycle of 60 units length. Identifications will also be possible in terms of subsidiary cycles, in this case of say 12, 15, 20, ... units, such subsidiaries deriving from combinations of the elementary waves taken two or more at a time. So that, even if some of the possible cycles are blurred, others may be manifest, leading to an identification in terms of values bearing some relationship to the elementary waves on the one hand, or to the major L.C.M. cycle on the other.

If the waves of the supposed pool are of equal probability, all having equal chances of emergence (the simplest assumption), then some L.C.M. cycles will be more probable than others. A cycle of 12 units length can be set up in quite a number of different ways out of wave-lengths 12, 6, 4, 3, 2 and 1 taken two or more together. On the other hand, a cycle of length 13 can only be set up in one way, namely by interaction between a wave of that length and one of unit length. From a relatively large pool of possible elementary waves, therefore, only a small number of major cycles need emerge, very minor cycles being relatively so infrequent as to be negligible.

How might such major L.C.M. cycles be distributed in the hypothetical pool of wave-lengths concerned? Taking the first two or three hundred natural numbers, and plotting each against the number of factors contained in it, Fig. 13*A* was obtained. Within the range of numbers shown, 180 is the most highly factorisable. Next comes 120. The number 60 is not very exceptional perhaps, but it is at least as heavily stressed as any in its immediate neighbourhood. On the whole, therefore, the series 60, 120, 180, ... covers all the major values. Other series are possible. Thus, for example, numbers 30, 60, 90, ... or 6, 12, 18, ... stand out relative to those surrounding them. And it should be noted that such series or chains interlace much as, a moment ago, we supposed chains to do among the logs of ratios.

Finally, each of the suggested chains of natural numbers has its origin in zero, the series running 0, 60, 120, ... 0, 30, 60, ... and so on. But this is exactly how series run among the ratios. The 0·0960 chain, for example, has a hypothetical first member in 0·0000.

Whence a new conception arises. In general arrangement, the empirically observed log wave-lengths seem to be scattered relative to one another much as are highly factorisable numbers in the natural scale, both in respect of relative magnitude, and also in respect of their origin in zero.

(*b*) *Cycle probabilities or frequencies: hypothetical and actual.*

The various series among the logs of the ratios can easily be matched among those of the natural numbers. Assuming that of

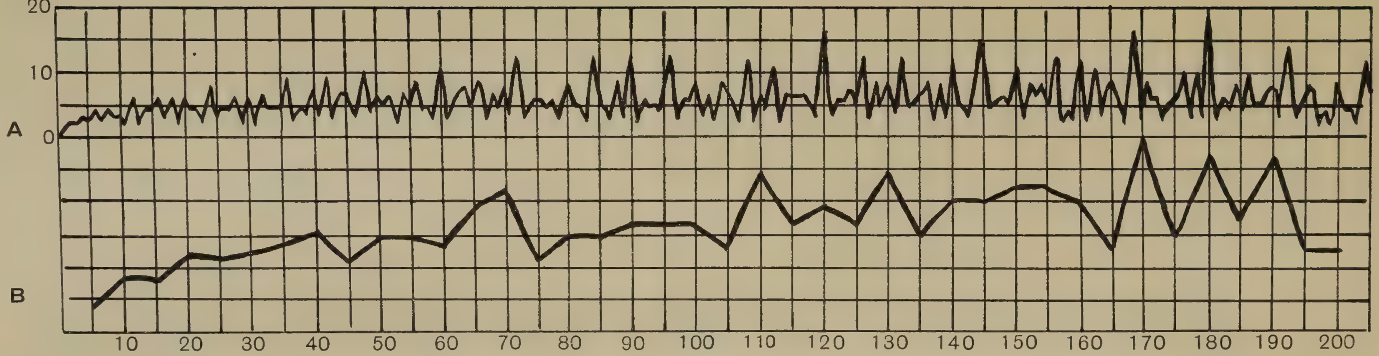

Fig. 13. A. Numbers of factors in the first 200 natural numbers. B. Figures of curve *A* pooled by fives and plotted against the centres of zones concerned.

Table III

Wave-length no.	Critical no.	No. of combinations	Logs		Ratio	Remarks
			Hypothetical	Empirical		
5	—	20	—	—	—	
10	—	60	—	—	—	
15	—	40	—	—	—	
20	—	100	—	—	—	
25	—	190	—	—	—	
30	30	270	0·0480	0·0480	1·116	
35	—	350	—	—	—	
40	—	380	—	—	—	
45	—	100	—	—	—	
50	—	750	—	—	—	
55	—	350	—	—	—	
60	60	3,500	0·0960	0·0951	1·245	
65	—	290	—	—	—	
70	72	2,800	0·1158	0·1158	1·30	
75	—	100	—	—	—	
80	80	850	0·1280	0·1271	1·34	
85	84	3,200	—	—	—	
90	90	3,400	0·1440	0·1440	1·39	
95	96	2,800	—	—	—	
100	—	200	0·1600	0·1584	1·44	
105	—	420	—	—	—	
110	108	3,600	0·1728	0·1732	1·49	
115	—	260	—	—	—	
120	120	50,000	0·1920	0·1920	1·55	
125	—	3,200	—	—	—	
130	—	3,400	—	—	—	
135	—	360	—	—	—	
140	—	3,300	—	—	—	
145	144	20,000	0·2304	0·2304	1·70	
150	150	3,300	0·2400	0·2360	1·74	
155	—	3,400	—	—	—	
160	—	3,200	—	—	—	
165	—	240	—	—	—	
170	168	49,000	—	—	—	
175	—	900	—	—	—	
180	180	160,000	0·2880	0·2880	1·94	
185	—	380	—	—	—	
190	192	11,000	0·3072	0·3010	2·03	
195	—	630	—	—	—	
200	198	5,660	0·3168	0·3200	2·07	
205	—	3,200	—	—	—	
210	210	51,000	0·3360	0·3345	2·17	
215	216	41,000	0·3456	0·3464	2·21	$(1{\cdot}49)^2$
220	220	3,400	0·3520	0·3522	2·25	$(1{\cdot}50)^2$
225	224	2,800	0·3584	—	2·28	$(1{\cdot}51)^2$
230	—	730	—	—	—	
235	—	3,200	—	—	—	
240	240	750,000	0·3840	0·3802	2·42	
245	—	350	—	—	—	
250	252	207,000	0·4032	0·4013	2·53	
255	—	470	—	—	—	
260	260	3,370	—	—	—	
265	264	50,000	0·4224	0·4267	2·64	
270	270	48,000	0·4320	0·4320	2·70	$1{\cdot}94^{\frac{3}{2}}$
275	—	860	—	—	—	

Table III (*continued*)

Wave-length no.	Critical no.	No. of combinations	Logs		Ratio	Remarks
			Hypothetical	Empirical		
280	280	11,800	—	—	—	
285	—	250	—	—	—	$1{\cdot}70^2$
290	288	150,000	0·4608	0·4608	2·89	
295	—	3,500	—	—	—	
300	300	222,000	0·4800	0·4800	3·02	
305	—	3,800	—	—	—	
310	312	53,000	—	—	—	
315	—	250	—	—	—	
320	320	11,200	—	—	—	
325	324	41,000	0·5184	—	3·30	$(1{\cdot}49)^3$
330	330	51,000	0·5280	0·5283	3·37	$(1{\cdot}50)^3$
335	336	800,000	0·5376	—	3·45	$(1{\cdot}51)^3$
340	—	6,000	—	—	—	
345	—	400	—	—	—	
350	352	6,000	—	—	—	
355	—	300	—	—	—	
360	360	13,000,000	0·5760	0·5760	3·77	$(1{\cdot}94)^2$
365	364	3,200	—	—	—	
370	372	3,200	—	—	—	
375	—	350	—	—	—	
380	378	52,000	—	—	—	
385	384	41,000	—	—	—	
390	390	10,600	—	—	—	
395	396	201,000	0·6336	0·6400	4·30	$(2{\cdot}07)^2$
400	400	20,000	—	—	—	

0·0960 to march with numbers 0, 60, 120, 180, ..., then at once those of 0·0480, 0·0160 and 0·0144 fall into line with numbers 0, 30, 60, 90, ..., 0, 10, 20, 30, ..., and 0, 9, 18, 27, ... respectively.

Matters are taken a step further in Table III, where enquiry is made into the number of ways cycles of given length can be built up. If, as assumed above, the elementary waves are taken to be equally probable, it follows that some L.C.M. cycles will be more probable than others, the chance of their occurrence varying according to the number of ways they can be constituted or built up. Stating the figures quite roughly (there is no need to go beyond a first approximation), "probabilities" were obtained for all cycle lengths from 1 to 400*.

Although such probabilities have been calculated for each cycle length separately, there is no need to state them all. Most of them are insignificant. The results have therefore been pooled. The several probabilities of cycles 3, 4, 5, 6 and 7 have been added together, and the result entered against their mid value 5 (columns 1 and 3 of Table III). And so for successive groups of five wave-lengths. If a very highly factorisable number occurs in one of the groups, it has been indicated in the column of "critical numbers." In such cases, it may be assumed that the number mentioned is responsible for the size of the "probability" value. The remaining columns need no description. Logs and ratios are set out in them assuming that the 0·0960 series of log wave-lengths actually does run with that of 0, 60, 120, 180, ... on the natural scale.

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* If a number has n factors, the number of ways in which a cycle of that length can be built up is 2^{n-1}, provided attention is confined to combinations in which the fundamental wave occurs with one or more of its harmonics. If it can further be set up by any two waves taken together (without the fundamental), there will be another 2^{n-3} cases. If a second pair of waves can be used (without the fundamental and the first pair), there will be an additional $3\,(2)^{n-5}$ cases, and so on.

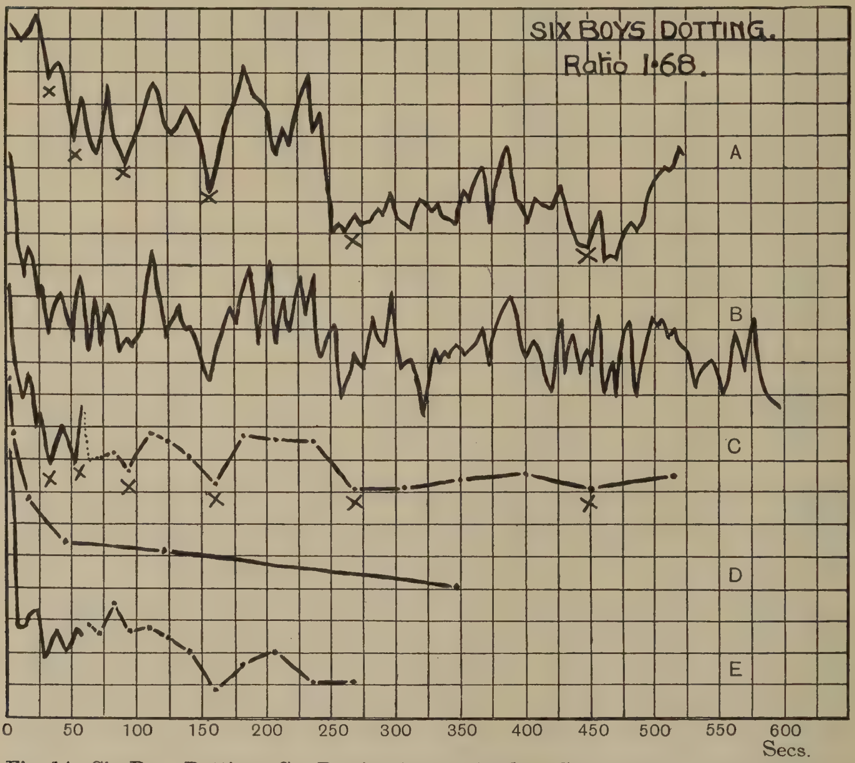

Fig. 14. Six Boys Dotting. Ga. Dotting (curve *E* only). Curves *A* to *D*: spells, output, quarter duration smoothing and double duration smoothing. Curve *E*: of quarter durations, intervals as for *C*.

Examining the table in detail, it cannot fail to be noticed how large some of the probabilities are. During the research, it has been a matter of surprise to find so few ratios. If the present analysis is sound, it might have been surprising to find more. This is especially so in certain regions. Ratios 3·02, 3·45 and 3·77 have stood alone. If the hypothetical frequencies or probabilities set against them are a true measure of the number of times they should appear, it is no wonder we failed to find intermediary values.

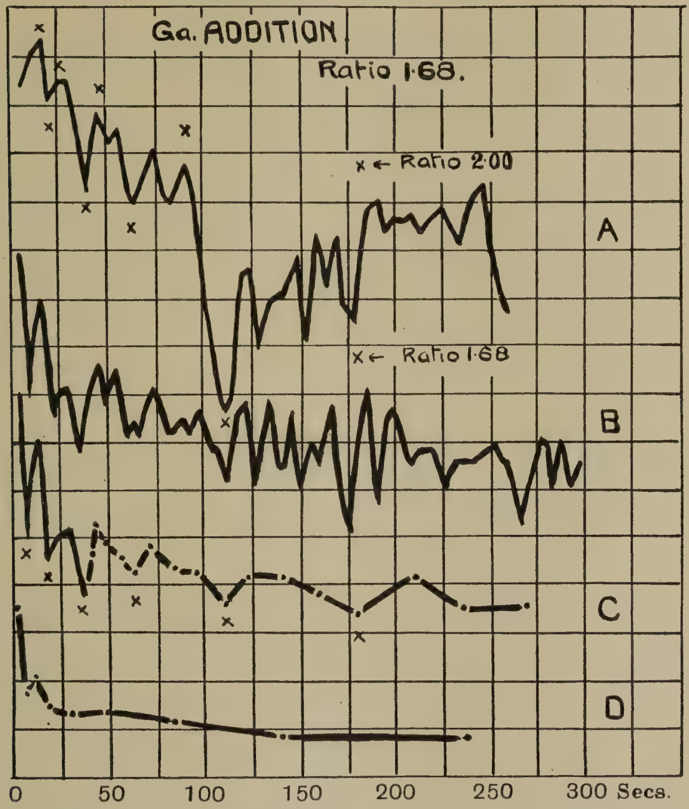

Fig. 15. Ga. Addition. Curves *A* to *D*: spells, output, quarter duration smoothing and whole (single) duration smoothing. Curve *D* given for comparison with Fig. 14*D*, illustrating the fact that double duration intervals are required to attain smoothness. Note that 15*C* is a quarter duration out of phase as compared with 14*C* and *E*. Later analysis: present example is more of ratio 1·70 (see Fig. 6); curves of Fig. 14 containing waves of hybrid wave-length (see Table VIII B).

Looking for ratios set against numbers of relatively low factorisability, there are found perhaps three worthy of note, namely 2·00, 2·09 and 2·70. The first two fall nearest to numbers for which the true equivalents would be 2·03 and 2·07. Even if the empirical ratios are amended in this sense, the probability of their occurrence is small as compared with other ratios in the neighbourhood. Remembering that all cases of nominal ratio 2·00 are suspect, and that we have only one case of nominal ratio 2·09, the verdict of the table may perhaps be accepted. We have not shown the two ratios to be of high probability in practice. On the other hand, ratio 2·70, the remaining ratio, has been of major importance, and the smallness of the theoretical frequency set against it has to be borne in mind.

Looking for large hypothetical frequencies against which no empirical ratios are placed, a case is found in wave-length 168, corresponding roughly with the 17th ratio of Table II. We have not measured or suspected such a value. Or again, a curious set of figures comes to light when seeking powers of ratio 1·50 (wave-length 110). In practice, all curves with waves of this order of size have been returned as of ratio 1·49 or 1·50, the difference (if any) between the two coefficients being regarded as one of chance. From the table, wave-length 108 has a higher degree of factorisability than 110. So that curves nominally of wave-length 110 may prove to be of wave-length 108 (i.e. of ratio 1·49 rather than 1·50). But at the third power level, emphasis is transferred to wave-length 336 (ratio $1{\cdot}51^3$), it having a probability much greater than those of wave-lengths 324 and 330 put together. The possible significance of this interplay between wave-lengths corresponding with ratios 1·49, 1·50, 1·51 and their powers will be discussed later.

There are then a few cases in which high probabilities remain (for the moment) with no empirical ratios against them, or in which there is doubt as to the way in which empirical and theoretical values should be paired together. On the whole, however, major ratios and major probabilities go together. And that is the essential point. By hypothesis, any possible cycle or wave-length might conceivably appear in one or other of the curves of the research. All that need be said therefore is that, with a limited number of curves, ratios of high hypothetical probability have appeared more often than not.

(c) *Of cycles in strong and weak curves respectively.*

Suppose we take a large number of single record curves, i.e. those based upon single attempts at a task. Suppose further, as an extreme case, that but one elementary wave appears in each. Since we may expect to hit upon all the said elementary waves sooner or later, there will be as many different curves as there are elementary waves.

But if we begin to add records together (or if more than one wave appears per record), then L.C.M. cycles will begin to appear. And since empirical identifications are in terms of such cycles, the range of possible measures will be decreased, for by hypothesis, some L.C.M. cycles are more probable than others. In other words, curves that are weak, statistically speaking, should tend to be more varied in type than strong ones. Evidence of this will be produced in Part II. Here we may note that evidence has already appeared in Tables I and II. The eight curves of Series II and III share five ratios between them, although their contributory curves gave us practically all the ratios of Table II. We have not been able to exemplify many of those ratios, simply because on pooling they have tended to disappear. The point is of importance, because quite directly it is evidence in favour of the general hypothesis. If we are right, then Table I ought to be narrower in range than Table II. Moreover, it ought to show a tendency to contract on curves of one dominant ratio, or powers thereof. It does so.

3. APPARENT PHASE RELATIONSHIPS

(*a*) *The problem.*

Problems of phase relationship have been troublesome. Wave sequences have seemed to be constant from curve to curve over quite a few examples, and then, as markedly, cases of apparent phase difference have occurred, usually of complete opposition.

There are at least three fundamental problems involved. Why should curves tend to agree in phase at all? It is true that we find constancy in wave-length, but that might come while phase relationships varied randomly. Secondly, when phase differences do come, why should they seem so often to be of complete opposition? This has been a characteristic calling insistently for explanation, one refusing to be brushed aside. Thirdly, why do waves of, say, ratios 1·30 and $1{\cdot}30^2$ (= 1·70) always run together, troughs of the major wave coinciding with every other trough of the minor one. Why, for example, cannot the two cycles lie athwart one another on occasion?

In general, it seemed that the facts might be explained by supposing all waves to come to a common trough somewhere in the curve. The present section is therefore devoted to a discussion of the consequences of such an assumption.

(*b*) *Inter-cycle relationships: the common trough point T_0.*

Let it be assumed that all the postulated elementary waves come together in a common trough somewhere in the curve, say at a point

T_0 seconds from the origin. In such a case, troughs of a 60 log unit cycle must occur at times T_0, $(T_0 + 60)$, $(T_0 + 120)$, ... units on the log scale. In similar fashion, a cycle of 120 log units will have troughs at T_0, $(T_0 + 120)$, $(T_0 + 240)$, ... and so on. In other words, its troughs must coincide with every other trough of the 60-unit cycle. Again, a cycle of 180 units will have troughs coinciding with every third of the 60-unit cycle, as well, of course, with those of the 120-unit cycle in places determined by their L.C.M. Put quite generally, therefore, it follows from the assumption of a common trough point that cycles must have troughs in common wherever possible, while when not coincident, adjacent troughs of any two given cycles must be distant by a multiple of their highest common factor. Conversely, of course, if there is no common trough, then the distribution of troughs will either be random, or follow some other system.

Before we can test this conception, it is necessary to arrive at accurate measures of cycle length. We have spoken of wave-lengths 60, 120 and 180, i.e. of very simply related whole number values. Our empirical measures are not as simply related as this. On the other hand, we have begun to assume that they actually correspond with the nearest whole number wave-lengths of Table III. Clearly, it will be as well to assume that table to be true for present purposes, and calculations are therefore made on that basis.

The next problem is of proportions, the fraction given wave-lengths are of one another. Wave-lengths 144 and 180, for example, stand in the proportion 4/5. That is to say, the one wave travels five complete cycles to every four of the other. Or again, wave-lengths 110 and 180 are as 11/18. So that when one is 18 cycles from the common trough point T_0, the other will be 11 cycles distant. Or we may calculate the length of the common cycle formed by any number of waves. The first two above mentioned have a common cycle equal to 4 periods of wave-length 180. The second pair similarly have a common cycle of 11 such periods. The combined cycle for all three must be the L.C.M. of 4 and 11, namely 44 periods. And so on.

This, then, was the method of handling the figures. The theoretical wave-lengths were assumed, and, taking 180 as the standard period, others were expressed as fractions of it, as in Table IV. From these, we arrived at the length of the common cycle any given wave should form with the standard wave.

As indicated in the example given above, it is required to find the L.C.M. of the numerators of given fractions. Numerators have therefore been entered separately under the heading of cycle indices. This term was used, because the values may also be employed when speaking of the ratios concerned. Thus, e.g., wave-lengths 144 and 180 are of ratios 1·70 and 1·94 respectively. Their common cycle, in terms of ratios, is of $1{\cdot}70^5 = 1{\cdot}94^4$. The index number 4, given against ratio 1·70 in the table, here appears as the power to which the standard ratio (1·94) is raised when measuring the said common cycle.

Table IV

Wave-length no.	Ratio	Fraction of 180	L.C.M. cycle index	Wave-length no.	Ratio	Fraction of 180	L.C.M. cycle index
60	1·245	$\frac{1}{3}$	1	234	—	$\frac{13}{10}$	13
72	1·30	$\frac{2}{5}$	2	240	2·42	$\frac{4}{3}$	4
84	—	$\frac{7}{15}$	7	252	2·53	$\frac{7}{5}$	7
90	1·39	$\frac{1}{2}$	1	260	—	$\frac{13}{9}$	13
96	—	$\frac{8}{15}$	8	264	2·64	$\frac{22}{15}$	22
108	1·49	$\frac{3}{5}$	3	267	2·67	$\frac{89}{60}$	89
110	1·50	$\frac{11}{18}$	11	270	2·70	$\frac{3}{2}$	3
112	1·51	$\frac{28}{45}$	28	280	—	$\frac{14}{9}$	14
120	1·55	$\frac{2}{3}$	2	288	$1{\cdot}70^2$	$\frac{8}{5}$	8
144	1·70	$\frac{4}{5}$	4	300	3·02	$\frac{5}{2}$	5
150	1·74	$\frac{5}{6}$	5	312	—	$\frac{26}{15}$	26
156	—	$\frac{13}{15}$	13	320	—	$\frac{16}{9}$	16
160	—	$\frac{8}{9}$	8	324	$1{\cdot}49^3$	$\frac{9}{5}$	9
168	—	$\frac{14}{15}$	14	330	$1{\cdot}50^3$	$\frac{11}{6}$	11
180	1·94	$\frac{1}{1}$	1	336	$1{\cdot}51^3$	$\frac{28}{9}$	28
192	2·03	$\frac{16}{15}$	16	340	—	$\frac{17}{9}$	17
198	2·07	$\frac{11}{10}$	11	348	—	$\frac{29}{15}$	29
204	—	$\frac{17}{15}$	17	352	—	$\frac{88}{45}$	88
210	2·17	$\frac{7}{6}$	7	360	$1{\cdot}94^2$	$\frac{2}{1}$	2
216	$1{\cdot}49^2$	$\frac{6}{5}$	6	384	$2{\cdot}03^2$	$\frac{32}{15}$	32
220	$1{\cdot}50^2$	$\frac{11}{9}$	11	396	$2{\cdot}07^2$	$\frac{11}{5}$	11
224	$1{\cdot}51^2$	$\frac{56}{45}$	56				

How now may we determine whether a common trough point T_0 actually exists? Turning to the curves of Fig. 6, it is found that ratios 1·70, 1·94 and 1·50 (among others) are present, waves of the cycles concerned all appearing to come to a trough at about the 900th second*. These cycles being of indices 4, 1 and 11 respectively, it follows that a common cycle of index 44 must be ending at that point. Whence

$$\log T_0 = \log 900 - 44x \log 1{\cdot}94 \qquad \text{......(1),}$$

where x is introduced because we do not know how many cycles of index 44 have elapsed since T_0.

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* Theoretically, they should either come together or be distant from one another by amounts depending upon their highest common factors. The minimum distance so determined would be of about 60 seconds. The observed differences are by no means as great as this, and have therefore been ignored. The 900th second has been taken as a round figure. There may be an error in so doing, but it hardly matters.

There are no obvious ratios of other indices in this set of curves. If the equation is to be taken further, it must be by bringing in examples in which ratios of other indices are to be found. But in so doing a difficulty arises. There are, for example, two curves of nominal ratio 3·02 (Figs. 11 and 12), one being apparently in opposition to the other. If they actually are of the same ratio, they cannot both have troughs at T_0. By trial and error, it was found that Fig. 11 will fit into a consistent scheme. The other was therefore put on one side for the moment*. Taking the trough sequence of Fig. 11*B* as an average result (the placings differ slightly in curves *A*, *B* and *C*, indicating how our empirical measures had varied from time to time), and assuming the wave-length to be of 300 units (index 5), we fall back upon the rule mentioned above that troughs of a wave of this log period should be either coincident with those of wave-length 180, or when not coincident, should differ by multiples of their highest common factor, namely 60. Now the troughs of curve *B* run 375, 125, ... seconds. Since 375 seconds is just 240 log units short of 900 seconds, this theoretical condition is fulfilled. Whence, the least possible alteration to equation (1) necessary to bring in ratio 3·02 is to make it run:

$$\log T_0 = \log 900 - 88y \log 1{\cdot}94 \qquad \text{......(2)}\dagger,$$

y being inserted, as x was before, because the result may still be indeterminate. Checking with ratios of other indices, it is found that all the major cycles are harmonised by assuming $y = 1$. For practical purposes therefore the equation can be cleared and solved to give

$$T_0 = 4{\cdot}076 \times 10^{-23} \text{ seconds} \qquad \text{......(3)}.$$

(*c*) *Trough sequences of the various ratios.*

(i) The argument. We are now embarked upon theoretical measures in terms of wave-length and phase. For having determined

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* Early attempts to solve this problem of inter-cycle relationships were baulked by these cases of apparent opposition. An additional difficulty was that we used empirical wave-lengths. This present attempt, using theoretical wave-lengths, was undertaken in the hope of finding a scheme that should cover at any rate most of the curves. When, however, the analysis was half-way through it became clear that, with it, we were going to be able to explain, or absorb, such cases of apparent opposition. It is an argument in favour of the analysis that, through it, we ultimately arrive at an interpretation of seeming anomalies.

† If in equation (1) we put $x=2$, then, since the index for ratio 3·02 is 5, the nearest common trough cycles of that ratio will make with those of ratio 1·94 is at

$$T_0 \times 1{\cdot}94^{90} = 900 \times 1{\cdot}94^2 = 375 \times 3{\cdot}02^2 \text{ seconds.}$$

The said common trough point is thus shown to be in the ratio 3·02 sequence assumed to pass through the 375th second.

a position for T_0, it is at once possible to calculate the trough sequence that should be characteristic of any given wave. Will such calculated series agree with the empirical ones, and if so, how far can agreement be taken as proof of the general hypothesis?

Four ratios, and the phase constants of two curves or sets of curves, were used in establishing T_0. They must be put on one side. Of the remaining cycles, the most that can be said is that their wave-lengths have entered into Tables II and III*. On the other hand, their phase constants most certainly do not enter into those tables, nor can they be derived from them as they stand. And since any given wave-length could be associated with many possible trough sequences, only one of which can be true as from the point T_0, it follows that if we can show that all our remaining trough sequences conform, it must be because the analysis is sound. We shall at last have fathomed the structure of the curve of fluctuations of attention.

Comparisons between empirical and theoretical results can be made quite simply. The 900th second is 88 cycles of the standard ratio from the point T_0. It may therefore be said to be of index 88, thereby indicating that all cycles of indices that are factors of that number should there come to a common trough. Thus, for example, ratio 2·07 is of index 11. Assuming it to be the equivalent of the empirical ratio 2·09, reference may be made to Fig. 6*A*, where its trough sequence is seen to pass through the 900th second.

Or taking another point in the curve, the 64th second is four waves of the standard cycle short of the 900th second, and so is of index 84. All cycles of indices contained in 84 (and there are many, it being so highly factorisable) must here come to a common trough. They are so numerous that the trough in question should be the deepest anywhere in the neighbourhood, in fact within the length of any of our curves—provided always that a representative sample of waves is present. The grand total curve of the research is probably representative in this sense (Fig. 28*A*). It is also sound, statistically speaking, being based on at least 4000 work units per 5-second interval. The trough that is relatively deepest in its outline actually falls at about 65 seconds. We have thus predicted the coming of a trough at a given point, and also its relative importance.

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* This, of course, has but little to do with the problem. We are not using the empirical wave-lengths at all, but the numbers of highest degrees of factorisability in the natural scale. The distinction is a very real one. As it happens, the comparisons now to be made would fail entirely if we calculated trough sequences corresponding exactly with empirical ratios. The analysis succeeds because we assume empirical ratio 2·40 to be an approximation to the theoretical 2·42, and so on.

(ii) Sequences of the ratio groups. For general purposes trough sequences have been calculated covering all the cycle lengths met with. They are set out in full in Appendix B. In this present section we may make a few extracts from that table, discussing sequences characteristic of the major curves met with.

Table V A

Wave-length	Ratio	Troughs (seconds)				
240	2·42 (2·40)	**26**	63½*	**154**	372	**900***
252	2·53 (2·52)	25	**63½***	160	**406**	1028*
270	2·70	23	**63½**	172*	**464**	

Second power troughs (i.e. of wave-length 480, etc.), in heavy type.
Third power troughs with asterisks.

Table V B

Examples (figs.)	Ratio	Remarks
4*C*	2·40 (2·42)	The standards to which other curves were referred.
4*D*	2·52 (2·53)	Second curve has signs of the half-duration wave.
19*B*	2·70	Originally returned as of ratio 2·40. See text.
10*A* and *B*	2·40	Two curves apparently in opposition. Points of approach or recession are of this ratio. Not necessarily a true wave (point at 900 seconds is of cross-over, not necessarily of crest or trough). Ratio concerned may be a power of the nominal one.
18*D*	$2{\cdot}70^3$	See text. Theoretical troughs at 9 and 172, empirically at 12 and 190 seconds.
18*E* and *F*	2·40 (2·52)	Case of apparent opposition.
20*A*–21*A*	2·70	Signs of ratio 2·70 between troughs at approximately 67½ and 180. Trough next following (nominally at 464 seconds) is less strongly marked in couple of cases, and seems usually nearer 480 seconds.
20*B*–21*B*		
21*D*		
21*E*		
21*F*		
31*G*		
31*H*	$2{\cdot}70^2$	(Fig. 31*H* is a smoothing of 21*F*)

The simplest family in some ways is that of ratios 2·40, 2·52 and 2·70. Calculated trough sequences are in Table V A (it being assumed that the nominal ratios are approximations to the theoretical values there mentioned). Theoretically, waves of all three ratios come to a common trough at the 64th second. Empirically, we made them begin at 62, 67 and 66 seconds respectively. The discrepancies are negligible compared with the size of a complete period.

By this finding, our chief experimental difficulty with the family is explained. We could only distinguish between its members (if at all) in terms of trough sequences, and it was often felt that the differences might be merely of phase. But if the sequences do come together at the 64th second, they will only become sufficiently divergent to be distinguishable from one another somewhere beyond the 400th second. Considering the shortness of some of the curves concerned, it is rather remarkable that we should have arrived at a threefold system of ratios at all.

The simplest cases are perhaps those of Fig. 20, where waves of ratio 2·70 are found in curves *A* and *B*, and in the total curve *D*. A similar example is that of Fig. 19*B*. This dates back to the period of indecision, and in practice ratio 2·40 was inferred, although had the curve been dealt with later we might have linked it with those of Fig. 20 as a case of ratio 2·70*.

Fig. 18*A* shows signs of ratio $2{\cdot}70^3 = 19{\cdot}70$ approx. This is the largest ratio suggested during the research. Key troughs appear at about $12\frac{1}{2}$ and 190 seconds†. Theoretically, they should fall at, say, 9 and 175 seconds. Considering the difficulty of estimating the centres of troughs even in curves much simpler than these, the discrepancies are negligible‡.

The ratio 1·49, 1·50, 1·51 group is more complex. It was shown, when discussing Table III, that although wave-length 108 is theoretically more probable than either 110 or 112, emphasis at the third-power level is transferred to wave-length 336 (ratio $1{\cdot}51^3$). We have therefore to unravel manifestations of both first and third-power cycles, remembering always that, at the moment, ratios 1·49 and 1·51

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* The smoothings of Fig. 19 have not been recast, for obvious reasons. Of the curve, as such, it may be noted that the trough theoretically expected at 64 seconds is not as well placed as it might be. For this, and other reasons, one would assume that the waves are of hybrid nature (cf. p. 55 *et seq.*). It is convenient to speak of curves as though they contain but one cycle apiece. It has, however, to be realised that each is based upon a complex of waves, a complex in which several major cycles may be fighting for dominance. As it happens, no one of the curves of nominal ratio 2·40 shown in this work can be said to be even relatively simple.

† In successive smoothings—curves *B*, *C* and *D*—these points seem to shift slightly. This, of course, is due to the placing of the smoothing intervals.

‡ It is interesting to note that no other ratio of this order of size has its troughs anywhere near the observed points. Thus, for example, by analogy with Fig. 6*A*, we might suppose that Fig. 18*D* is of ratio $2{\cdot}09^4$ or $2{\cdot}07^4$. Theoretically, however, troughs of those ratios would come at 375 and 19, or 900 and 48 seconds approximately, relatively long distances from the empirical points.

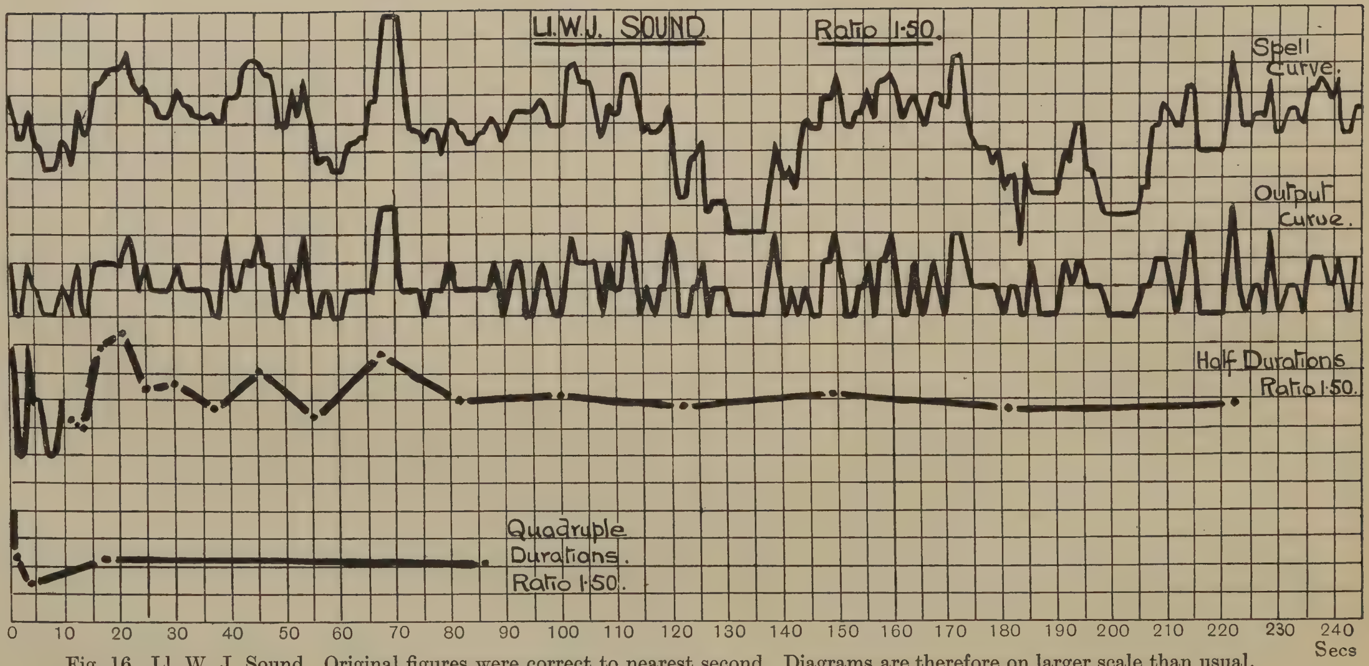

Fig. 16. Ll. W. J. Sound. Original figures were correct to nearest second. Diagrams are therefore on larger scale than usual.

are of purely theoretical import, all empirical measures being in terms of ratio 1·50 or its powers.

From Table VI A it is seen that the calculated trough sequences of the theoretical ratios 1·49 and 1·51 come to a common trough at 64 seconds, thereafter diverging. On the other hand, waves of ratio 1·50 should here be at a crest, thus being in apparent opposition. On the whole, therefore, we may fail to distinguish between curves of the two theoretical ratios in this zone—and for some distance thereafter—but we should at least be able to separate them from cases of the true ratio 1·50.

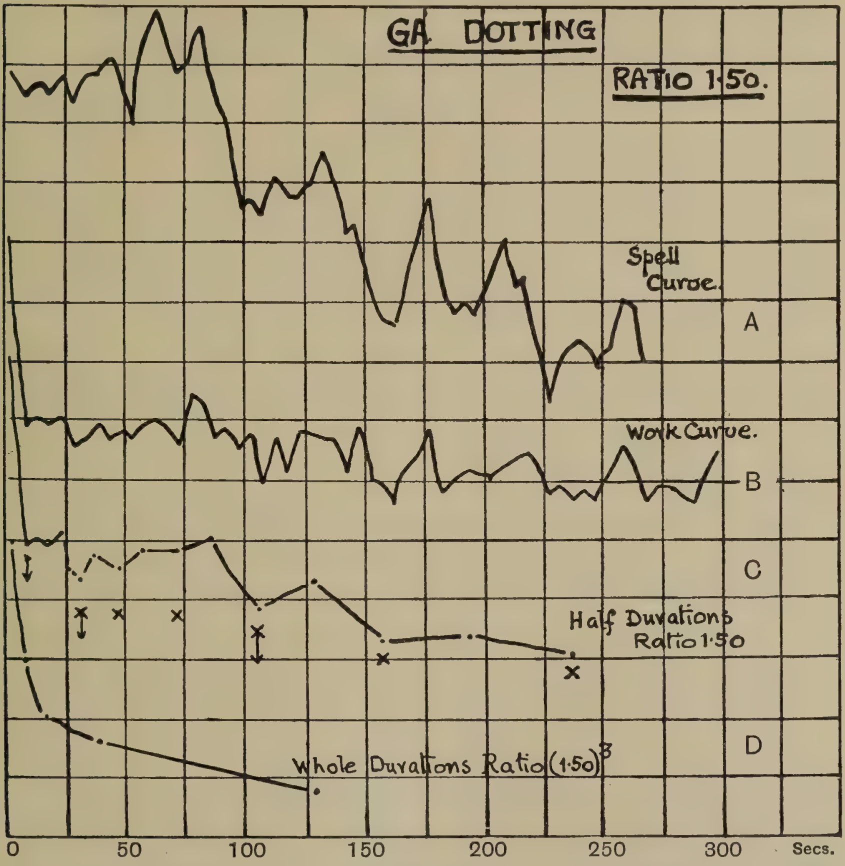

Fig. 17. Ga. Dotting. Note that the material of Fig. 14*E* is contained in the present example.

Curves to which the nominal ratio 1·50 has been attributed are

Table VIA

Wave-length	Ratio	Troughs (seconds)
108	1·49	__63½__ _94*_ __141__ 209 __312*__ 464 __690__
110	1·50	53 __79*__ 119 __178__ _267*_ __400__ 600 __900*__
112	1·51	__63½*__ 96 __145__ 219* __330__ 499 __754*__

Second power troughs (i.e. of wave-length 216 etc.), in heavy type.
Third power troughs with asterisks.

Table VIB

Example (fig.)	Ratio	Remarks
6_B_	$1{\cdot}50$, $1{\cdot}50^3$	The first-power cycle of this curve was used in establishing T_0. Third-power cycle not so used. Credit can be claimed for fact that its troughs conform to the table.
8_B_	$1{\cdot}51^3 = 3{\cdot}46$	See text.
12_B_		
16	?	Too short to be determinate. Possible wave of ratio $1{\cdot}49^5 = 2{\cdot}70^2$. (Troughs at 2, 8 and 60 seconds. Theoretically 1¼, 9 and 64 seconds.)
17_C_	$1{\cdot}49$, $1{\cdot}49^3$? $1{\cdot}42$, $1{\cdot}42^3$?	Troughs approximately as for either ratio. See Table C below.
21_A_	$1{\cdot}49$, $1{\cdot}49^5$, $1{\cdot}51^3$	See text.
21_B_		
21_C_	$1{\cdot}50$ ($1{\cdot}50^2$?)	All three cases are of interest in that cycle only becomes clear towards end of curves. May therefore be composite cycle. (See discussion of Table IX.)
_E_		
_F_		

Table VIC

Fig. 17_C_, actual troughs at	_7½*...27_	42	72	_107*_	160	240
Wave-length 108 (ratio 1·49)	_8½*...29_	43	63½	_94*_	141	209
Wave-length 96 (ratio 1·42)	_13* ...37_	53	76	_108*_	154	219

listed in Table VIB. Taking first those of Fig. 21*, we had empirically divided them into two groups, one apparently in opposition to the other. One group has troughs running 900, 600, 400, ... seconds.

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* Examples of Fig. 21 may perhaps seem rather complex on first sight. It may be wondered why given ratios were attributed. In practice, they happened to be given to other forms of the curves. Thus, for example, ratio 1·50 sequences are characteristic of the spell curves concerned. (These are not shown for reasons of space.) Ratio 2·70—to be mentioned in a moment—is characteristic of some of the work curves when drawn against ordinary arithmetic time intervals. (See Fig. 20_A_ and _B_, where the curves of Fig. 21_A_ and _B_ are translated from log time units to ordinary ones. The difference in appearance is rather striking.) Note should further be made of the fact that the curves are of totals per series. They must contain all the waves met with in the contributory curves concerned, and it is easy to understand why they should be difficult to read in terms of single cycles.

These are obviously as for the theoretical ratio 1·50. The other group has troughs running approximately 720, 480, ... down to 63 seconds. Such values lie roughly midway between corresponding ones on the theoretical ratio 1·49 and 1·51 scales. A moment ago it was shown that we ought to have confused cases of these ratios. We seem to have done so. The empirical ratio for Fig. 21*A* is a mean value; its trough sequence might have been compounded of the two calculated ones. Theoretical speculations could hardly be more completely justified, for, to repeat, we have sought for signs of these two ratios on purely theoretical grounds.

How now do third-power cycles lie? Here difficulties arise. There seems to be one example each of ratios $1{\cdot}49^3$ and $1{\cdot}50^3$*, but of $1{\cdot}51^3$, theoretically the most probable, we have at first sight no very obvious examples—as far as triple loopings of the minor cycle are concerned. It will be shown in a moment that there may be one or two distorted examples, but, to begin with, it is perhaps as well to point out that higher power cycles of this kind are not necessarily in terms of multiple loopings of the minor cycle. It is a fault of our terminology that this is suggested. Waves of ratio $1{\cdot}51^3$, for example, should be envisaged as long simple waves of that size, rather than as triple groups of smaller waves.

Thinking, then, of waves of 336 log units length, and comparing with other theoretical sequences, it becomes apparent that, for short curves, there is but little difference between the trough sequence of this wave-length and that of wave-length 360. They have a common trough at 64 seconds, and in our short curves, at any rate, there is great danger of confusing the two theoretical cycles. A possible example is that of Fig. 8*B*. It might be of either wave-length, or both†. A related curve, Fig. 12*B*, is similar, having a sharply defined trough at about 70 seconds, followed by a long hump in the curve that might be of several wave-lengths, the said hump making the curve look somewhat like a sickle, or a letter S laid on its side—a characteristic outline.

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* Table VIc shows that ratios 1·42 and $1{\cdot}42^3$ may be present in Fig. 17 instead of, or as well as, the nominal ones. The curve is too short to say more. Something of the same kind may be happening in Fig. 6*B*, where ratios 1·55 and $1{\cdot}55^3$ could conceivably be present (both having a common trough at 900 seconds with first and third-power cycles of ratio 1·50). In this respect it is interesting to note that the said curve smooths quite well in terms of ratio $1{\cdot}94^2$ ($1{\cdot}94^2 = 1{\cdot}55^3$). Cf. Fig. 31*B*.

† Trough sequences run as follows:

Empirical curve ... 20, 70, 215, 240 ... seconds.
Wave-length 336 ... 18, 63½, 219 ... ,,
Wave-length 360 ... 17, 63½, 239 ... ,,

The material of this curve is contained in that of Fig. 12*B*. If we had earlier noted the resemblance between the *B* (output) curves, we might not have returned Fig. 12*A* (spell curve) as being of ratio 3·02. See, however, further analysis of this example on p. 62.

Fig. 21*A* is of this same sickle shape. Rising out of a deep depression at the 67th second, it falls over into a series of six rather roughly formed waves of the hybrid ratio 1·51/1·49 type. They come in loops of three, of which only the first is at all shapely, the other being flattened by a trough at about the 480th second. The threefold grouping is of the ratio $1{\cdot}51^3$ type.

A cycle of ratio 2·70 seems to be present, as indicated by troughs at (approximately) 480, 180 and 64 seconds. (See also corresponding curve, Fig. 20*A*. It is curious how strongly marked the ratio 2·70 cycle is when the curve is expressed against ordinary time units.) Waves of this ratio should have a common cycle with those of ratio 1·49 between 64 and 464 seconds, five waves of the one going with two of the other ($2{\cdot}70^2 = 1{\cdot}49^5$). This seems to be so (Fig. 21*A*), although the empirical markings on the curve, being of nominal ratio 1·50, do not fit as well as they might.

Summing the curve up, we thus find it to be of the sickle-shaped type, with (*a*) single waves of the hybrid ratio 1·49/1·51, (*b*) apparent threefold waves of ratio 3·45 ($1{\cdot}51^3$)*, and (*c*) a single five- or twofold looping of ratio $1{\cdot}49^5 = 2{\cdot}70^2$. The description is of interest because it applies to Fig. 21*B* as well. As the two curves between them cover about two-thirds of the material of the research, we may here be approaching the outline of what may be termed the standard work curve.

Cases of nominal ratio 1·70 and 3·02 need not be tabulated. The standard wave-lengths have been taken to be of 144 and 300 respectively. Corresponding trough sequences will be found in Appendix B. Only two of the examples presented in this work were empirically returned as being of ratio 3·02 (Figs. 11 and 12). Of these, one has already been removed as being more of ratios 3·45 and 3·77 ($1{\cdot}51^3$ and

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* If ratios 1·49 and $1{\cdot}51^3$ are most probable at first and third-power levels respectively (Table III), it follows that a complex and statistically weighty curve of this kind may have strong waves of those two ratios present together with, but heavily swamping, weaker ones of ratios 1·51 and $1{\cdot}49^3$. In actual fact the triple cycle does seem to be of the $1{\cdot}51^3$ type.

It may analogously be asked why the first-power cycle is not more clearly of ratio 1·49. Here, however, there would of necessity be interference, at every third trough, with the strong ratio $1{\cdot}51^3$ cycle. The curve is not long enough to show whether such interference becomes less marked as we get away from the common trough at 64 seconds.

$1{\cdot}94^2$). The other (Fig. 11) conforms to the theoretical trough sequence—in fact, was used in the determination of T_0—and so needs no further comment*. Of cases of ratio 1·70, first and second-power troughs conforming to those of Appendix B may be found in Fig. 15, first, second and fifth-power troughs in Fig. 6. The other curves of this nominal ratio (Fig. 14) are apparently anomalous, and will be dealt with later.

Trough sequences of the ratio 2·00 family, expanded to cover the "missing" 17th ratio of Table II, are set out in Table VII A. The said missing ratio would seem to be of wave-length 168. But this is the half duration of wave-length 336 already dealt with under the guise of ratio $1{\cdot}51^3 = 3{\cdot}45$. It was there shown that there might be confusion with wave-length 360 (ratio $1{\cdot}94^2$). The trouble is likely to be as great with the half-duration cycles as set out in the present table. It is no wonder that we have no empirical identifications in terms of the said 17th ratio. It might have been evidence against the general hypothesis had we made any†.

Ratios 2·07, 2·03 and 1·94 form a triad analogous with that of the ratio 1·51 family. As there, the two outside values come to a common trough (this time at 900 seconds), the middle one having a crest at that point. It follows that cases of apparent opposition may be expected. A possible case is found in Fig. 10. It should be noted, however, that there is association with ratio 2·40. The curves are not as simple as they might seem, and are dealt with again in the next section.

The examples empirically returned as being of this family (Table VII B) need little further comment. It should, however, be noted that the curves are complex. Thus, for example, there are signs of several ratios in the curves of Fig. 9, while those of Fig. 7 have many of the characteristics of the ratio 3·02 system*. It is a peculiarity of this group that they have been characterised by a crest sequence (at 45, 90, 180, ... seconds) rather than by troughs, as

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* Fig. 7 *B*, nominally of the ratio 2·00 group, has troughs at approximately 38 and 128 seconds. These are more characteristic of ratio 3·02 (see corresponding points in Fig. 11 *B*, noting moreover the triple hump lying between them in both curves). Were the curves being assessed anew, we might therefore return them as both of ratio 3·02. Incidentally, their outline in the 38–128 seconds zone is quite different from that of curves of the sickle-shaped type (cf. Figs. 8 and 12). The cases are illustrative of the difficulty of analysing curves as short as these, although, in the light of the present analysis, they do tend to show how we may develop the notion of types.

† See p. 62 for a further analysis of Figs. 11 and 12, and possible confusion of wave-length 168 with wave-length 150 as well as with 180.

Table VIIA

Wave-length	Ratio	Troughs (seconds)
168	1·86	34 **63½*** 119 **219** *406** **754** 1400
180	1·94	33 **63½*** 123 **239** *464** **900**
192	2·03	**37*** 76 **154** *312** **632** 1281
198	2·07	**49** 101 **209*** 434 **900**

Second power troughs in heavy type.
Third power troughs with asterisks.

Table VIIB

Ratio	Remarks
1·86 and $1{\cdot}86^2$	See ratio $1{\cdot}51^3$ (Table VI).
1·94	Standard smoothing is of Fig. 6*C*.
$1{\cdot}94^2$	Best sought in tendency to double humping in curves of nominal ratio 2·00. See Figs. 5*C*, 7*C*, 9*C*, 31*B*. Of these, 31*B* is contained in 5*C*. (Fig. 22*F* looks as though it might be exceptional, troughs not coming as they should. Curve is too short to say what is happening. Ratio 2·70 may be more characteristic.)
$1{\cdot}94^3$	($1{\cdot}94^3 = 2{\cdot}70^2$.) Fig. 31*H*, also possibly 21*A* and *B*. Fig. 31*H* is a smoothing. The original curve (21*F*) does not manifest this wave as much to the eye.
$1{\cdot}94^4$	($1{\cdot}94^4 = 1{\cdot}70^5$.) Fig. 6*D*, *E* and *F*. The twentyfold loop ending at the 900th second would be of this ratio.
$2{\cdot}03^2$	Fig. 14*E* and possibly 14*B*. Troughs at 37 and 154 seconds.
$2{\cdot}03^3$	($2{\cdot}03^3 = 1{\cdot}70^4$.) Fig. 15*B*. Troughs at 37 and 312 seconds.
2·07	Apparently only one case, namely Fig. 6*A*. Dotted line under curve seems to indicate presence of a second power wave, but if so, it is opposed in phase to the theoretical sequence for such a cycle. On smoothing, the said apparent cycle disappears.

N.B. Half-duration smoothings of ratio 1·94 (cf. Figs. 22 and 31), are of intervals centred at presumed crests of the ratio 1·39 ($=1{\cdot}94^{\frac{1}{2}}$) wave. The said intervals are thus a quarter duration from where they would have been had we centred them on troughs.

with curves of other ratios. Now that a complete systematisation in terms of troughs has been made, the said crest sequence might perhaps be abandoned. In fact, it becomes suspect. In the next section, for example, it will be shown that crests at 45 and 90 seconds may be cut off by troughs in which waves of many ratios meet. They are not necessarily characteristic of any one cycle. Or again, it has already been pointed out that crests expected at 22½ or 180 seconds may be weakly marked or absent (pp. 18 and 22). On the whole, therefore, identifications in terms of the crest sequence are not likely to be as sound as those based on troughs. The empirical findings have, however, been allowed to stand.

(*d*) *Common trough zones: hybrid cycles.*

Considerations now arise that lead, in many ways, to a new conception of the curve. We shall see why some of the experimentally determined curves may seem to have one wave-length at the beginning, another near the end. Or again, we arrive at a fresh notion of the problem of apparent opposition in phase.

Beginning with a simple example, the curve of Fig. 6*A* has six well-marked minor crests between, say, 38 and 155 seconds. They run three to a complete wave of ratio 2·07, and so may be said to be of ratio $2{\cdot}07^{\frac{1}{3}}$. When the curve was thought to be more of ratio 1·94, they were correspondingly returned as being of ratio $1{\cdot}94^{\frac{1}{3}}$. Now that there are theoretical trough sequences available, it might seem that the problem could be decided. Of which ratio are they? Curiously enough, they might be of either. Moreover, as shown in Table VIII A, the sequence could be of at least half a dozen minor cycles ranging in length from, say, 60 to 72 log units*.

The curve is not an exceptional one. Five such cases are quoted in Table VIII B, and others might have been given. Several problems therefore arise. Why so many curves of the kind? Why this particular portion of the curve (for the manifestations die out by about the 300th second). Why cycles of this particular wave-band?

The portion of curve in question is centred on the 64th second. This is a well marked common trough point. If, therefore, there are any waves of the 60–72 wave-band present, and if they come to a common trough in this zone, they will run together for some distance, thus setting up a zone of enhanced amplitudes, a zone in which the apparent wave-length will be dependent upon the average wave-length of the cycles concerned. Outside that zone, of course, amplitudes will tend to become more and more damped, signs of the said average cycle becoming at the same time indeterminate as the various waves tend to interfere more and more with one another.

We here for the first time touch upon average periods, as compared with L.C.M. cycles. Taking the simplest possible case, a curve of but

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* Table VIII A has on the left certain major ratios and their corresponding wave-lengths. Next come troughs characteristic of those waves in this zone, and finally the number of minor waves lying between those troughs, with consequent inference as to their wave-length in terms of the major cycles concerned.

Table VIII A

Major wave-length	Ratio	Troughs (seconds)	Inference as to minor wave
240	2·42	63½–154	Four crests present. Wave-length 60
252	2·53	63½–160	Four „ „ „ 63
192	2·03	37–154	Six „ „ „ 64
198	2·07	49–101	Three „ „ „ 66
270	2·70	63½–172	Four „ „ „ 67½
288	2·88	37–108	Four „ „ „ 72

Table VIII B

Example (fig.)	Wave-length (approx.)	Stretch within which wave is manifest
6*A*	66	37–154 seconds.
14*A* 14*C*	70½	52½–270 seconds. Spell curve and smoothings. Show rather abrupt breakdown of wave at latter trough.
14*E*	70½	50–160 seconds. Five fairly well marked minor waves before the breakdown point is reached.
15*C*	72	37–140 seconds. Five waves. Curve not long enough to show whether breakdown is permanent.
21*F*	67½	37–280 seconds. Eight waves. Curve then merges into one of ratio 1·50.

two waves, of 4 and 5-unit periods respectively, may show marked "beat" effects. If the waves come to a common trough, then at every such point there will be a zone of increased amplitudes, as described above, corresponding zones of damping lying midway between them. These zones will be 20 units apart, and measures of that distance ought to be fairly accurate. Suppose, however, that we only possess a short length of such a curve, say a piece 10 or 15 units long centred about a common trough zone. In such a case, our attention is likely to be taken by the average period of the waves at their point of maximum amplitude. We may overlook the L.C.M. cycle altogether, saying that the wave present is of, say, 4½-unit period, such a measure, of course, being of the average period within that zone.

Two kinds of measures are then possible, the one of fairly straightforward and accurate L.C.M. cycles, the other of average or composite periods derived from common trough (or common crest) zones. And this is true, whether of a simple case such as the one mentioned, or of the most complex curve met with during the research. On the whole, it seems clear that the major ratios of Tables II and III have been of

L.C.M. cycles. But some of our measures may have been of the composite kind, being taken from curves too short to get us sufficiently far away from the marked common trough zone near the 64th second.

The problem next arises why we should so often have met with cycles of the 60–72 wave-band in that zone. Tables IX A and IX B are of trough sequences characteristic of the most important cycles within the 54–75 and 72–100 bands respectively. It has been convenient to give them with the usual signs indicating troughs of higher power waves as well*. Suppose now that we have a curve containing all or most of these cycles. In the curve they will be present as a complex. Here they are, as it were, analysed out. Can anything be inferred from the table as to the nature of the curve as a whole? Clearly, as we already know, there will be a marked trough at the 64th second, most of the cycles, whether of first or higher powers, there coming to a common trough. The next most commonly occurring trough point is at the 900th second, although the cycles are not necessarily the same as those concerned at the earlier one. Some, e.g. that of 60 units, have a trough at both points, others at one only. Nevertheless, conditions should be much the same in both zones.

To what extent will they be similar? The question is best answered by pointing to a very important difference. Five out of the seven first-power cycles of Table IX A come to a common trough at 64 seconds. Whence it follows that any curve complex enough to carry signs of all these cycles will have a well-marked composite wave of this order of size in the 64-second zone. Reinforcement of wave by wave will be at a maximum. Interference will begin as soon as the 64th second is left behind—that is obvious—but in the neighbourhood of that point it will be at a minimum. With waves of Table IX B, conditions are less favourable. Four first-power cycles come to a common trough here, but three are in opposition. To that extent, apparent waves of this order of size will flatten out. Granted, therefore, a curve complex enough to show signs of all these possible cycles, the chances are

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* These sequences were chosen by taking the same set of fundamental waves (of ratios between 2·21 and 3·02), working up to them from their fourth and third harmonics respectively. Most of the major cycles are thus covered, in fact we are reclassifying them.

It has been necessary to bring in a half integer value ($67\frac{1}{2}$) to complete the table. Such a cycle is theoretically impossible as long as the unit of Table III is assumed (0·0016). The sequence of troughs given for it would, however, become possible if the unit were halved. In such a case all the nominal wave-lengths would be doubled, corresponding ratios of course remaining unaltered.

Wave-length 72 occurs in both tables, being the fourth harmonic of one major cycle, the third of another. This duplication of values does not affect the main arguments.

Table IX A

Basal wave-length	Ratios 1st, 2nd, 3rd, 4th powers	Troughs (seconds)
75	1·32 1·74 2·29 3·02	_54_* __71__ 93 __123__‡* 162 __214__ _282_* __372__‡ 490 __646__* 851 __1122__‡
72	1·30 1·70 2·22 2·89	49 __63½__* 83 __108__‡ _141_* __183__ 239 __312__‡* 406 __529__ _690_* __900__‡
67½	1·28 1·64 2·11 2·70	49 __63½__‡ _81_* __104__ 134 __172__‡* 220 __282__ _362_* __464__‡ 595 __763__* 977
66	1·27 1·63 2·07 2·64	_62_* __79__ 101 __129__‡ 164 __209__ _267_* __340__‡ 434 __553__* 706 __900__‡
63	1·26 1·59 2·00 2·53	50 __63½__‡ 80 __101__ _127_* __160__‡ 202 __255__* 322 __406__‡ _512_* __646__ 815 __1028__‡*
60	1·24 1·56 1·94 2·42	51 __63½__‡* 79 __99__ _123_* __154__‡ 191 __239__* 298 __372__‡ _464_* __578__ 721 __900__‡*
54	1·22 1·49 1·82 2·22	_52_* __63½__‡ 77 __94__* 115 __141__‡ _171_* __209__ 255 __312__‡* 380 __464__ _565_* __690__‡ 966

Table IX B

Basal wave-length	Ratios 1st, 2nd, 3rd powers	Troughs (seconds)
100	1·44 2·09 3·02	59 __85__‡ _123_* __178__ 257 __372__‡* 537 __777__ _1122_*
96	1·42 2·03 2·89	53 __76__ _108_* __154__‡ 219 __312__* 444 __632__‡ _900_*
90	1·39 1·94 2·70	46 __63½__‡* 88 __123__ _172_* __239__‡ 333 __464__* 646 __900__‡
88	1·38 1·91 2·64	_49_* __67__‡ 93 __129__* 178 __246__‡ _340_* __471__ 651 __900__‡*
84	1·36 1·86 2·53	47 __63½__‡* 86 __119__ _160_* __219__‡ 298 __406__* 553 __754__‡ _1028_*
80	1·34 1·80 2·42	__47__‡ _63½_* __85__ 114 __154__‡* 206 __279__ _372_* __499__‡ 670 __900__*
72	1·30 1·70 2·22	49 __63½__* 83 __108__‡ _141_* __183__ 239 __312__‡* 406 __529__ _690_* __900__‡

Second power troughs in heavy type.
Third power troughs *.
Fourth power troughs ‡.

rather more in favour of waves approximating to those of wave-band *A*; rather less in favour of wave-band *B**. At the 900th second, however, emphasis transfers to Table IX B. This time, reinforcement is more likely with waves of that order of size. We get therefore the very interesting finding that, theoretically speaking, it should be possible to find complex curves such that the dominant cycle is apparently of the *A* type in the 64-second zone, of the *B* type in the 900-second zone, the said cycles being, however, of hybrid nature since, by hypothesis, they are set up by the interaction of several, or many, cycles of the given order of size.

That then is the problem. Empirically we know that cycles of wave-band *A* do often occur in the 64-second zone. Is it also a fact that curves of the kind may have a cycle of wave-band *B* in the other zone? It seems to be so. From about the 300th second onwards, the 66-unit cycle of Fig. 6*A* tends to fade out, giving place to one of about 90 units period. This is even better seen in Fig. 6*C*, where the 90-unit cycle has troughs at approximately 320, 450, 650 and 900 seconds, with a possible further trough of the same sequence beyond that point†.

Time has not permitted the examination of other long curves‡. The sum total of all we have is, however, available (Fig. 21*F*). It is not an independent case, but it shows that the addition of fifteen other records to the ten of Fig. 6*C* does not destroy the manifestations, although it slightly alters the wave-lengths. There is a cycle of about 67½ units length in the one zone, of about 110 units (ratio 1·50) in the other §.

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* Many cycles, long and short alike, come to a trough at one or other of the points 38, 49, 64, 105, and, say, 155 seconds. Although not exactly sequential, they conform roughly to a cycle of about 66 units length, and the presence of these quasi-independent troughs in a curve may therefore suggest, or emphasise, a cycle of that order of size. There is here an additional argument going to show why wave-band *A* may seem to be dominant in that zone. Incidentally, the grand-total curve of the research (Fig. 28*A*) has troughs at 38, 64, 105 and 155 seconds approximately. The troughs are obviously of importance.

† The cycle in question is a half duration of ratio 1·94, troughs of which are indicated on the curve.

‡ See, for example, Fig. 18*A*, in which three waves (of period something within wave-band *A*), occur between 64 and 105 seconds. Before anything could be said of the other end of the curve it would, however, be necessary to smooth it.

§ There is here an explanation of a puzzling feature of this curve. It is so definitely of ratio 1·50 at the end (even when unsmoothed as in Fig. 21), that one could not understand the absence of waves of that length near the origin. Incidentally, the tables ought to be re-cast, bringing in waves up to, say, 120 units in wave-band *B*. As they stand they do not cover the present curve. (Its significance was not realised until, the present argument being elaborated, we turned to it to see what was happening in the two zones.)

The available curves thus strongly suggest that cycles of the given bands may be expected in the two zones more often than not. The problem needs, however, to be more fully explored.

The present analysis throws light on a long-standing problem. Curves of Fig. 14 (ratio 1·68 or 1·69) failed to conform when the ratios were first tabulated (p. 31). Even with the more elastic system of Table III, they were apparently anomalous, being of wave-length 141—a highly improbable value*. Marked signs of a half-duration wave that, in our present nomenclature, would be of wave-length 70½, were also to be found. Here at last there seems to be an adequate explanation of such measures. They are quite possibly of composite cycles of interference manifestations within the 64-second zone.

As a final problem, we may take that of apparent opposition in phase with curves of nominal ratios within the range 2·21 to 3·02 (i.e. that of the fundamental cycles of Table IX, given as fourth and third powers in the *A* and *B* sequences respectively). Directing attention to Table IX B, it is seen that of the seven sequences, four have their third powers coming to a trough at 64 seconds, the other three being roughly in opposition. The same kind of thing happens at the 900th second, although the grouping of cycles is not the same.

Suppose now, as an extreme case, that we have two work curves, one containing the four cycles that come to a trough at the 64th second, the other containing the three that do not. They should clearly be of much the same average ratio, but opposed in phase. The same applies to curves based on cycles that do or do not come to a common trough at the 900th second. Extreme cases of the kind are not likely to happen, but there may be tendencies that way. In other words, cases of apparent opposition may occur with curves of ratios of this order of size, opposition being either in the 900 or the 64-second zones. Moreover, from the fact that cycles of the two kinds are fairly evenly

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* By trial and error, the best-fitting wave-length was found to be of 0·2260. Whence we get wave-length 141, since 141 × 0·0016 = 0·2256. In rather extraordinary fashion, the curve is found to conform to the 141 sequence in the matter of phase as well (compare with given sequence, Appendix B). 141 is an unlikely wave-length, since it has but four factors, namely 1, 3, 47 and 141. It would be improbable even in a curve based upon but one record, still less so in one of 69 records (Fig. 14, curves *A* to *D*) or even of 8 (Fig. 14*E*). Moreover, if we admit it here, we might have to bring it in for other curves, e.g. Figs. 6*A* or 9*E*. It is only by this conception of hybrid manifestations in a zone of interference that we escape a dilemma. We cannot postulate hypothetical probabilities, and then put forward a number of cases of one improbable wave-length.

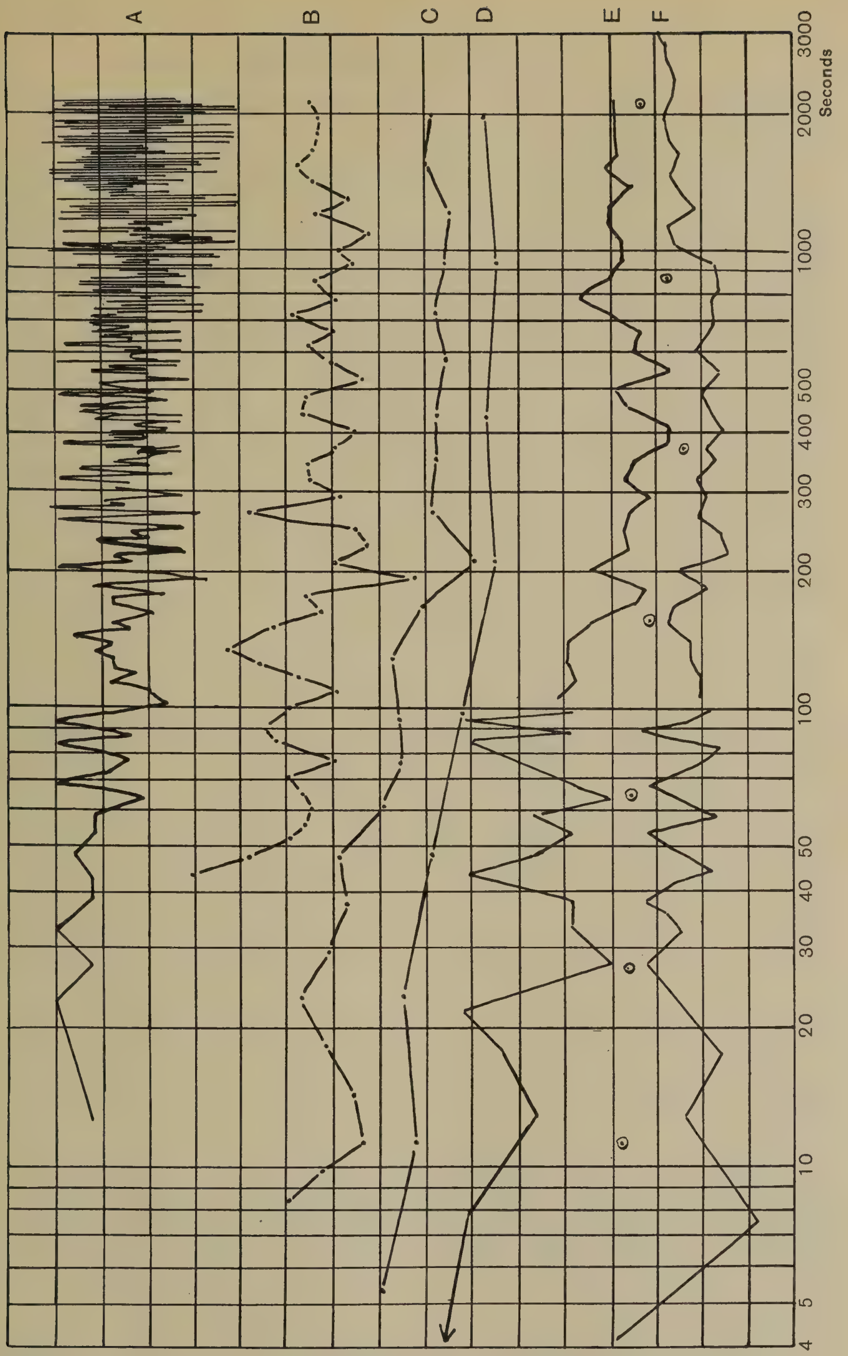

Fig. 18. Series IV: Three examples. Curves *A* to *D*, Stepney Division; *E*, Stepney Multiplication; *F*, Pe. Multiplication. Output figures in *A* are of the original 5-second intervals. In *B*, *C* and *D*, they are of averages over log intervals of 0·0360, 0·1080 and 0·3240 respectively. In *E* and *F*, they are of 0·0400, save that the original 5-second intervals are used for the first 100 seconds. Marks between *E* and *F*, indicating points of approach, are of the ratio 2·40 sequence. (See Figs. 4*C* and 10*A* and *B*.)

distributed in the tables, we may infer that such cases should be of frequent occurrence[*].

There are only a few curves long enough to cover the 900th second, but of them at least two pairs can be brought into account, namely, Figs. 10*A* and *B*; 18*E* and *F*. Points of approach and recession are marked in terms of the empirical ratio 2·40 sequence, although in neither pair does that ratio fit as well as it might. (It need not, of course, if the cycles are of composite origin.) In the one pair, opposition is most marked from the 900th second onwards. In the other, it as clearly fades by the 130th second. The curves seem to be representative of opposition in the two zones. And once more the analysis throws light on a puzzling feature of the research. That there is opposition in the two pairs of curves was undoubted. But it was not easy to say why it should be localised so definitely, and yet so differently.

The remaining cases are of the 64-second zone. Figs. 11*A* and 12*A* were made to be of ratio 3·02, albeit opposed in phase. The case is not of complete or mirror oppositeness, the *C* curves concerned being apparently normal as far as half-duration waves are concerned (ratio $3{\cdot}02^{\frac{1}{2}} = 1{\cdot}74$). Opposition is of the pairing of the said minor waves, one system of grouping being seen in Fig. 11*C*, another in Fig. 12*C*. Compare now theoretical trough sequences of ratios $3{\cdot}02^{\frac{1}{2}}$ and $3{\cdot}46^{\frac{1}{2}}$[†]. The said troughs differ by seconds within the length of curve covered. But with the major cycles, opposition is at once apparent. And it is of the kind met with in practice, namely of the pairing of minor crests. In this case we have been fortunate in being able to speak of definite wave-lengths. In other cases, e.g. those of ratio 2·40 above, it is only possible to speak generally, saying that this kind of thing is to be expected.

As a final example, Fig. 9*E* is nominally of ratio 2·67, although it might be of anything from 2·64 to 2·70, the measure being rough.

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[*] Account ought, of course, to be taken here of the relative probabilities of the cycles concerned. This may perhaps be taken to depend upon the number of factors in the wave-length numbers, (cf. Fig. 13*A*). These do not differ much, as far as the given cycles are concerned, and the problem has been ignored. (We are not, for example, dealing with figures as large as those of Table III. The question there was of the number of ways in which the said factors can be combined—when taken two or more at a time—to form cycles of the given lengths.)

[†] Wave-lengths 150 and 168, found by taking second-power troughs of wave-lengths 75 and 84 in Tables IX A and IX B respectively. The major cycles are of fourth powers. Wave-length 180 might also have been involved as well as, or in place of, 168.

Seeking a theoretical equivalent, it is seen that whereas troughs of the half-duration cycle fall much as they should for a ratio $2{\cdot}70^{\frac{1}{2}}$ sequences (wave-length 135) the pairing of minor crests is opposed to that for a true ratio 2·70 cycle. The case is probably on all fours with the last one.

Reference may finally be made to the grand total curve of the research, Fig. 28*A*. By hypothesis, it is of all the elementary waves met with, of all the L.C.M. cycles, of all the composite waves just dealt with. It has a strongly marked trough at 64 seconds, less strongly marked ones at 38 and 105 seconds. A copy of Fig. 6*A* has been brought in below it to show that troughs of the hybrid 2 × 66 unit cycle march with those just mentioned. The curves are related, although one is based upon over 600 records, the other upon but six.

The two crests cut off by troughs at 38, 64 and 105 seconds are centred at, say, 50 and 90 seconds. We thus finish the analysis where it began, namely with the familiar 45, 90, 180, ... sequence of crests, the last named, as so often, being weakly marked or absent.

Taking the curve as a whole, it is of the S or sickle shape mentioned on p. 52. Rising sharply out of the trough at 64 seconds, it falls over the slope of the sickle towards its handle, just as though scores of waves were pouring out of that point, up over their first following crests, into a zone where crests and troughs fall into inextricable confusion. It moves just as it should if our analysis is well founded, although with so involved an outline we should never have dared to guess at its meaning, save in the light of all the steps leading up to the present conception.

(*e*) *Tentative criticisms.*

When the preceding section on phase relationships was drafted, the analysis therein described was but a few weeks old. The problem as such had often exercised one's mind, and the final attempt on it was undertaken to illustrate the kind of thing envisaged, but from which we had seemed to be barred by occasional phase trouble. Having determined to get the best-fitting scheme for curves that would fall into line, and noting the possible significance of such theoretical ratios as the 1·49, 1·50, 1·51 triad, the trouble of apparent phase discrepancies was found to roll up and disappear as we advanced.

Table III is of theoretical "probabilities," assuming a unit of 0·0016, with ratio $1{\cdot}94^2$ set over against the dominant wave-length of the range covered. Table IV is of wave-length proportions and cycle indices. As such, it is of pure numbers, and independent of the actual wave-lengths and unit of Table III. The said unit could be changed in many ways (e.g. by halving it) without affecting the cycle indices of Table IV, and in consequence, without affecting the tables of trough sequences, these latter, of course, being determined essentially by the cycle indices of the ratios concerned*.

On the other hand, the one alteration we have so far considered would affect matters by cutting out some of the theoretical ratios altogether. It derives from the fact that ratios 2·70 and $2{\cdot}70^2$ $(= 1{\cdot}94^3)$ seem to be more important in practice than would be indicated by the probabilities of Table III. It might, therefore, be worth while to see what would happen were the tables recast, making $1{\cdot}94^3$ the dominant ratio in place of $1{\cdot}94^2$. The simplest (although not the only) way of doing this would be to set ratio $1{\cdot}94^3$ against wave-length 360, altering all the others to conform. This would not affect the Table IV indices for ratios remaining in the list, but some would disappear, as e.g. 1·50 and 1·51 at first and second-power levels. They would still, however, appear as third-power values, thus saving our analyses, since it is with their third powers that we have had to deal in the main†. On the other hand, ratio 2·40 would be set against wave-length 160, one of low probability, and there are other con-

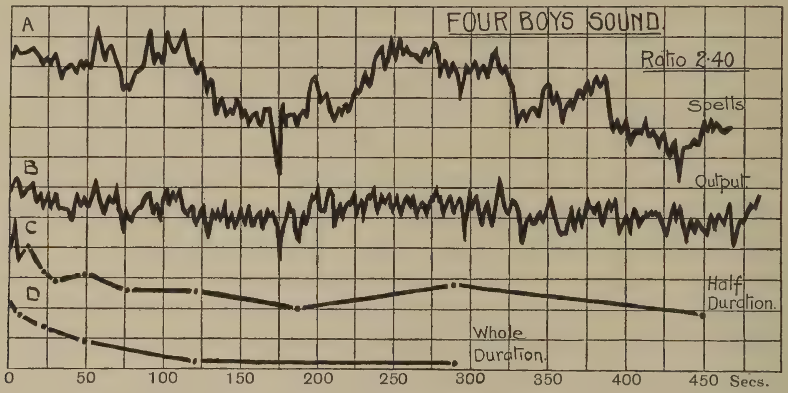

Fig. 19. Four Boys Sound. Later analysis: Example may be more of ratio 2·70. See Fig. 20.

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* That is to say, the calculated trough sequences of given ratios remain unaltered, even if the unit is changed. All that happens is that the theoretical "wave-lengths" are doubled if the unit is halved, and so on.

† There are no definite examples of first or second-power cycles of ratio 1·51. Those of ratio 1·50 are apparently associated with what we have called hybrid or composite cycles. If the pure ratios are assumed not to exist at these lower levels, we should have to attempt an explanation of all such manifestations in terms of composite cycles.

sequences not yet fully explored. It is not clear at the moment whether anything would be gained by altering the system in this particular way, but it is as well to mention the possibility. By so doing, emphasis is laid upon the general utility and soundness of the present scheme while yet indicating that it is not necessarily final.

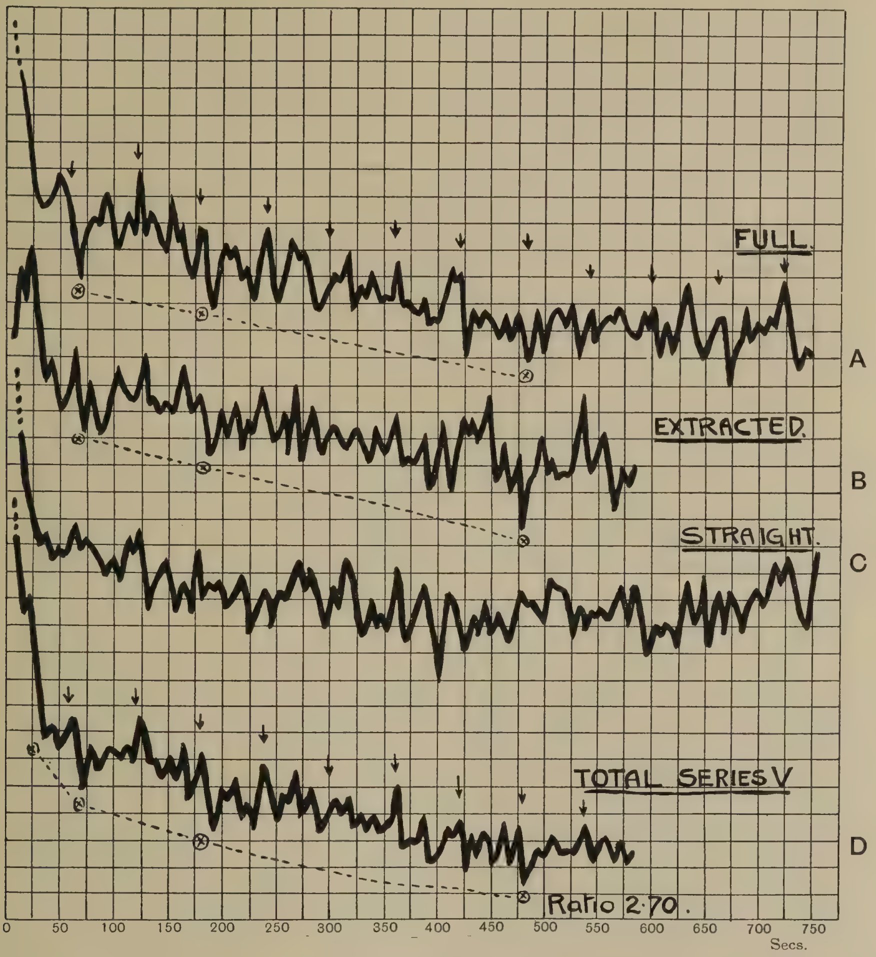

Fig. 20. Series V: Total output curves. Note that these curves are all of output. From this point onwards, the *A*, *B*, *C*, *D* convention of earlier examples (spells, output, part and whole duration smoothings) is dropped. The meaning of terms Full, etc., is given in Part II.

The great problem is whether apparent emphasis upon ratio 2·70 is due to dominance of a true L.C.M. cycle of that size, or to the fact that it happens to be the most characteristic "composite" cycle within the length of our curves.

The strong point in favour of Table III, with its emphasis upon ratios 1·94 and $1{\cdot}94^2$, is that it derived simply and naturally from Table II. At the same time, there appeared to be contributory evidence in that curves of the ratio 2·00 group are so numerous (assuming, of course, that the 45, 90, 180, ... second sequence is basally of ratio 1·94).

But after the finding of ratio 2·70 in the total curves of Series IV and V (Figs. 20 and 21) the ratio 2·00 group seemed to become less dominant, while with the final analysis in terms of theoretical trough sequences, the 45, 90, 180, ... crest sequence was dealt a final most severe blow, it now appearing to be not of an L.C.M. cycle at all, but a summation effect within a common trough zone. (For that is how one reads Fig. 28*A*, mentioned above.)

By this same final analysis, however, ratios 2·52, 2·53, 2·67 and 2·70 have themselves been shown to be too often associated with manifestations of apparent opposition in phase, or of summation, for many of the examples to be of pure L.C.M. cycles. In fact, they form the group within which such manifestations are most characteristic. And since we can explain them in terms of Table IX, which itself depends upon Table III, it follows that the system should not be interfered with too hastily.

It is not easy to say why curves should have the same common trough point T_0. There is here a difficulty to be met when visualising a psychophysical mechanism that could work that way. As they stand, the findings are purely empirical. It may further be pointed out that without constancy in phase we should not have been able to draw up general purposes trough sequences. Without those tables of troughs, we should not have been able to cross-check empirical measures of wave-length (and phase) with theoretical ones. The case for periodicity tends to stand or fall with that of constancy in phase.

It has not been possible to measure the limits within which empirical trough sequences can be said to conform to the theoretical analysis. We have but few cases of any given ratio, which makes it difficult to strike an average. For present purposes, we may perhaps turn to a set of curves in which apparent discrepancies strike one as being large. In practice, troughs of ratio 2·70 have been placed at points lying between 445 and 480 seconds as compared with 465 on the theoretical scale. In log measures, the range is of 0·0743 as compared with a wave-length of 0·4314. The total range is thus about a sixth of a period, or plus and minus a twelfth about the theoretical point.

A better way of looking at the problem perhaps is to turn to the grand-total curve of the research (Fig. 28*A*), enquiring what happens at the 64th second. Long cycles coming together here will tend to make the common trough wide. Smaller ones will narrow it. Any straddling the point will tend to fill it up. On the whole, therefore, we should here get a trough of definite width, although how wide it might be we cannot say. Over and above such factors of a regular nature are those of chance errors and the like*. These would presumably tend to flatten the curve. In actual fact, the trough in question is of marked depth, and of no great width. It does not look as though there has been any very great tendency for individual records to vary.

4. CONCLUSIONS

Logarithms of the empirical ratios fall into series. The first tabulation was of common increment 0·0480, having its origin in zero. Other series are possible. Most of the major cycles are covered by one of 0·0960. Practically all ratios of any weight are covered by one of 0·0160. In all cases, origin is in zero. It is inferred from these findings that there is a unit of wave-length. For practical purposes the unit has been taken to be of 0·0016, although even that value may be relative only. Dividing through by this unit, all wave-lengths have been expressed as natural numbers. These we have used as measures.

If there are many elementary waves in a curve, our empirical measures of wave-length will probably be of complex L.C.M. cycles, rather than of simple waves. Such L.C.M. cycles will be of lengths corresponding with highly factorisable numbers. Using the numbers 1 to 400, it has been shown that the major log wave-lengths are actually scattered, much as are the most highly factorisable numbers in the natural scale.

Assuming constancy of phase, and further that all possible cycles come to a common trough point T_0 somewhere in the curve, it is calculated that $T_0 = 4{\cdot}076 \times 10^{-23}$ seconds from the origin. From this point, theoretical trough sequences have been determined for all the wave-lengths met with. The characteristics of two curves and four cycle lengths were used in the calculations. Comparing the remaining examples, it is found that empirical and theoretical trough sequences agree, and that within fine limits. In particular, cases of the following ratios have been examined:

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* Chance errors, not only of moment to moment variation but also of errors in timing. If, for example, the subject begins mental work a second before the nominal time of starting, there will be a constant error of that amount throughout, our 64th second being the 65th in reality, and so on.

(1·94) $1{\cdot}94^2$ $1{\cdot}94^3$ $1{\cdot}94^4$ (1·70) $1{\cdot}70^2$ $1{\cdot}70^3$ $1{\cdot}70^4$ $1{\cdot}70^5 = 1{\cdot}94^4$
2·42 2·53 2·70 $2{\cdot}70^2 = 1{\cdot}94^3$ $2{\cdot}70^3$
2·03 $2{\cdot}03^2$ $2{\cdot}03^3$ 2·07 (3·02)

while, splitting curves of nominal ratio 1·50 into three groups, we get further:

1·49 $1{\cdot}49^3$ $1{\cdot}49^5 = 2{\cdot}70^2$ $1{\cdot}51^3 = 3{\cdot}46 = 1{\cdot}86^2$
(1·50) $1{\cdot}50^3$

ratios in brackets being those used in establishing the tables.

The 900th and 64th seconds are 88 and 84 cycles of ratio 1·94 distant from the point T_0. For practical purposes those points can be said to be of indices 88 and 84, and so for any other point in the curve. There is here a systematic nomenclature* for the curve, one from which the trough sequence of any given wave-length can be directly inferred.

Cases of apparent opposition in phase seem to be completely explained. Within given lengths of curve, sequences characteristic of wave-lengths of approximately the same size can often be confused, especially in the neighbourhood of the 64th and 900th seconds, points where large numbers of waves should (theoretically) come to common troughs.

By thus arriving theoretically at trough sequences for all the known ratios, cross-checking empirical measures of wave-length with theoretical measures of phase, we seem to have substantiated our main contention, namely that work curves are periodic, that there are many elementary waves present, their logarithmic periods being whole-number multiples of a given (but as yet unmeasured) unit of logarithmic time, that the said unit is universally true for all our subjects, under all conditions of experiment met with, and that finally, true phase differences from curve to curve seem to be negligible.

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* Cf. Appendix B.

Part II

SPECIAL PROBLEMS

A. INTRODUCTION

THE findings of Part I raise at least five questions of outstanding importance. They are:

(*a*) apparent absence of individual differences,
(*b*) apparent absence of specific differences,
(*c*) apparent absence of phase differences,
(*d*) constancy in the unit of log wave-length,
(*e*) geometricity as such.

By chance, imagining geometricity in the curves (and one can imagine many things from a curve), we might have arrived at a ratio for each individual curve. But those ratios need not have been related to one another. Or granted a system of relationships, it need not have been of this strangely curious type. Why should all men be equal in respect of wave-lengths? Why should the nature of the task be immaterial? Why should there be no phase differences? What is the underlying reason for a unit of wave-length? There are difficulties in every direction. Taking the manifestations as a whole, there seems to be too great an element of invariability.

A full and complete way of retreat would be found if it could easily be shown that the subjects had developed corresponding imposed rhythms, proving, for example, that invariability in the results is due to the use of special time signals or the like. Unfortunately, such an explanation raises almost as many problems as it settles, problems too of the same order or kind. Two people sitting side by side, listening to the same time signals, etc., may give, one a curve of ratio 2·00, the other of, say, ratio 1·70. On another occasion, they may change over, or produce yet other ratios again. There is no predicting what they will do. But since the cycles they do produce have ratios falling into a series, etc., we say there are no individual differences. It is, however, only in respect of the unit of wave-length. In details of outline, and so on, there are wide differences. Any theory in terms of imposed rhythms would have to be framed to show how they could be made manifest in this somewhat complex fashion.

The fact that waves seem to be geometric makes it difficult to imagine an origin in imposed rhythms. Those usually suggested, e.g. of time signals or the like, are of arithmetic or constant period. It is necessary, therefore, to enquire into the relationship between possible arithmetic cycles and the geometric ones of the research.

In saying, a moment ago, that there are no individual differences, reference was being made to the unit of wave-length. It is not that other differences are absent or impossible. It is conceivable, for example, that some subjects may tend to use certain elementary waves more often than others. A cycle of, say, ratio 1·94 can be set up in many ways. Its existence in the curves of subjects *X*, *Y* and *Z* does not prove that they each arrived at it in exactly the same way. Nor does it prove that they drew upon identically similar "pools" of possible wave-lengths.

Or again, let it be supposed that subjects tend to use longer (or shorter) wave-lengths when fatigued. It does not follow that this would alter the apparent ratio of their curves, provided always that changes are by multiples of the unit of wave-length. If, for example, the original waves in use are of lengths 1 to 200, and if by fatigue all are lengthened by 1 unit, the apparent result will be as for a pool ranging in length from 2 to 201. The major cycles are not likely to change.

It has just been assumed that changes must be by multiples of the unit. This is implied in the general hypothesis. If changes could be by infinitesimal degrees, the whole system of relationships established in Part I would break down. If the system holds as we have interpreted it, it follows that even if individual and specific differences, fatigue effects, and so on exist, it will be most difficult to measure them, simply because they must take place by definite amounts. Were variations to occur by infinitesimal degrees they would be obvious; occurring by multiples of the unit, we may not be able to detect them.

A possible line of research would be to collect a large number of curves to see if those from different people or tasks, etc., tend to occur in definite bands of the wave-length scale. As an example of the kind of thing meant, curves of subject Sm. fall on one side of Table I, those of subject Ga. on the other. We have regarded this as being a matter of chance. It may not be.

In this second part of the work we shall have to do with some of these special problems. In the first section, that of possible general and specific waves, enquiry is made as to the way in which groups of curves are scattered in respect of certain combined measures of wave-length and phase. Later sections have to do with possible waves of arithmetic (or constant) periodicities (whether natural or imposed), with fatigue measures, and so on. Finally, an attempt is made to show how the findings may be said to bear upon other problems of psychophysics.

B. GENERAL AND SPECIFIC FACTORS: SIMILARITY BETWEEN CURVES

1. The problems.
2. Experimental work.
3. Data and findings.
    - (*a*) General argument: nomenclature.
    - (*b*) Work curves and smoothings.
    - (*c*) Average shapes.
    - (*d*) Structure of shapes.
    - (*e*) Individual differences.
    - (*f*) Of geometricity and phase in Extracted curves.
4. Conclusions.

1. THE PROBLEMS

Waves could conceivably be specific to the tasks performed, general in the sense that they appear whenever mental work is being done, or common in that they appear with more than one task, although not with all. By the findings of Part I, "ratios" are neither specific to the task nor general. In a broad sense, we shall not be able to go behind that finding. On the other hand, granted the assumption that individual ratios (or the wave cycles measured by them) can be set up by many different combinations of elementary waves, it follows at once that specific, common and general elements could exist (whether generating waves of identically the same length or not) and that without the fact becoming apparent in ordinary work curve analysis. If, for example, curves from Task *A* and Task *B* are both of ratio 2·00, all that can safely be said is that the two combinations of waves have the same L.C.M. This proves or disproves nothing, least of all that there is nothing specific to either task.

In attacking the problem, we therefore tried first to break up given complexes, attempting to isolate waves of different origin, hoping even to be able to produce such curves as that of "waves specific to the act of Addition" and so on. This proving to be impossible, we fell back upon arguments to be drawn from the way in which our attempted curves of this nature scatter themselves in respect of certain combined measures of wave-length and phase.

2. EXPERIMENTAL WORK

Series V was undertaken with about 100 boys in the upper classes of an L.C.C. Elementary School (Bow, E.). Most of the work was sandwiched or alternated. The subjects began work upon one task (the *A* task), but at a given signal, changed immediately to another kind of work (the *B* task). If it was necessary to use another kind of paper for the *B* task, the two sheets were pasted together, the join acting as a change signal. If one sheet would serve, a red line was drawn across at the desired point.

To ensure that any losses due to the act of changing should be evenly distributed and not fall at one point in the curve, the various sheets were graded, some children changing at the 60th second, some at the 65th, and so on until the 150th second, by which time all had changed. This also ensured the mixing of phase constants, a somewhat important point.

As comparisons were to be made with ordinary or straightforward work curves, a few records of that kind were collected from some of the children. (Comparisons were actually to be made with curves of the earlier series, but these present records were required as a control group.)

Five tasks were used, Addition, Subtraction, Multiplication, Division, and Dotting, the last-named as a check on the others in case their common nature as arithmetical performances should cause trouble. It was impossible to ensure that every child should make an attempt at each of the twenty-five different arrangements of the tasks (twenty sandwiched; five straightforward). Each child was therefore allotted two tasks, and kept to one or other of the combinations possible with them.

Just enough practice was given to make sure that the instructions were understood. The subjects then made four attempts at one or other of the possible combinations of the two tasks allotted to them (including performances with single tasks for the ordinary work curves required). The tests were made on two successive school days, two attempts per day with an interval of 15–20 minutes during which scripts were collected and fresh ones given out. After rejecting obviously incorrect work (scripts in which time intervals had not been properly kept), there remained some 250 sandwiched and about 50 straightforward records.

3. DATA AND FINDINGS

(*a*) *General argument: nomenclature.*

We thus tried to separate waves peculiar to one of two tasks from those common to both. For if any wave is common, it will run through the double record; if peculiar, it will start or stop at the point of change. Waves peculiar to the first or *A* tasks cannot be isolated. They occur in normal relationship with any possible common waves. Those peculiar to the second or *B* tasks, being quite abnormally placed in this respect, may perhaps be separated from the complex and separately identified.

Curves showing sandwiched work in full will be called Full curves. They presumably contain all the waves that, on given occasions, have been common to two sandwiched tasks. Curves showing output figures for the corresponding *B* tasks (cutting out the *A* portions altogether, and renumbering the time intervals to make the point of change zero time) will be called Extracted curves. In them should be found waves that, on the given occasions, have been peculiar to the *B* tasks alone. Any ordinary or straightforward curves will be called Straight curves*.

Individual sandwiched curves will not be presented (i.e. curves in which a given *B* task has been combined with any given *A* task). The sum of all the Extracted (or Full) curves in which Addition has been the *B* task will be called the Extracted (or Full) Addition curve. Curves simply labelled Full, Extracted, or Straight, without further qualification, are of grand totals for the kind of material in question summed without regard to the nature of the tasks performed.

(*b*) *Work curves and smoothings.*

As we have just said, curves for the various combinations of tasks are not shown. The only safe thing to be said of them is that they vary among themselves much as do contributory curves of Series I–IV. No particular ratio can be said to be general, i.e. appearing in all Full curves, none can be said to be specific to Extracted curves of given tasks.

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* Following Phillips(14) we might have called them general, specific, and normal respectively. Our curves are not quite of the same nature as his, and for that, as well as other reasons, it has seemed better to adhere to the more non-committal names used during the research.

The three grand-total curves are given in Fig. 20, and again in Fig. 21, against ordinary and log time measures respectively. A set of smoothings of half durations of ratio 1·94 is also presented (Fig. 22). Others were calculated but are not given. There is no apparent need for them. By hypothesis, the curves are complex enough to contain most of the possible L.C.M. cycles, and it would be difficult to get

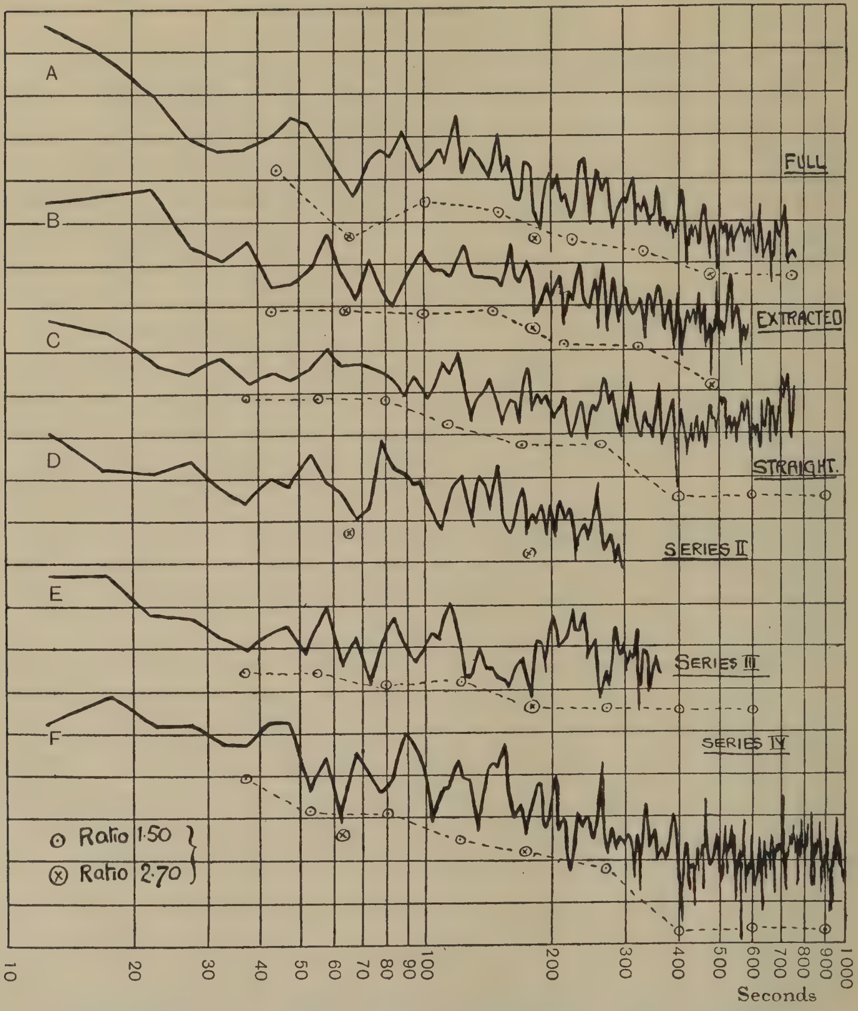

Fig. 21. Series II to V: Total output curves. Plotting is against the log of time. Curves *A*, *B* and *C* are as for Fig. 20.

consistent smoothings in terms of any one cycle. Grand-total curves for the other series are given in Figs. 21 and 22. They are included because, although not strictly comparable in weight (or even in length), they are yet tending to approach the Series V pattern in some of their general characteristics.

The Extracted curve of Fig. 20 differs from the others in that it begins with a slight rise extending over the first 25 seconds. This is, however, followed by a distinct fall off, and we are probably justified in saying that there is a decrement in Extracted curves, much as with ordinary or Straight curves, save that it may be interfered with somewhat by disturbances attaching to the act of changing from one task to another*. Apart from this, there is little or nothing to distinguish the said Extracted curve from the Full one just above it, whether as shown here, or in Figs. 21 and 22. Both are of the sickle type mentioned on p. 52, and after comparing them, whether in the many smoothings undertaken, or in the form of "shapes" to be described in a moment, one was forced to conclude that the curves are more alike than different. In other words, the sum total of all the waves that might conceivably be specific tends to be of the same major L.C.M. cycles as the total of all the waves that might conceivably have been general (or common to the tasks taken two at a time). Can we now, from such a result, decide whether there are such things as general, common, or specific waves?

Strictly speaking, no single wave should be said to be either specific or general until all the possible combinations of tasks have been examined, an impossible task. Despite this ban on individual identifications, signs of generality or of specificity may be sought by enquiring how Full and Extracted curves ought to behave in given circumstances. The possibilities are as follows:

(i) If only general waves exist, none being specific or common, then all Full curves should resemble one another and also all Straight curves. They should all be of one general pattern. On the other hand, Extracted curves should be meaningless (i.e. non-periodic), except in so far as they might be interpreted as the end parts of corresponding Full curves. Remembering, in so interpreting them, that big phase differences were set up by the method of experiment (children changing tasks, some at 60 seconds, some at 65 seconds, etc., from the origin), it follows that an Extracted curve should, on this supposition, be very definitely damped, and very definitely out of phase, as compared with corresponding Full or Straight curves.

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* In changing from the *A* to the *B* task, manifestations of inertia are to be expected. These may show in a stoppage of work, or in the making of mistakes. Thus, e.g. children often broke back into the *A* task, even after doing two or three sums of the *B* type, and so on. The rise in the curve may thus be due to the overcoming of momentary inhibitions and the cutting out of errors.

(ii) If there are only specific waves, individual Extracted curves should vary with the task performed, resembling corresponding Straight curves in all respects. Full curves should be meaningless.

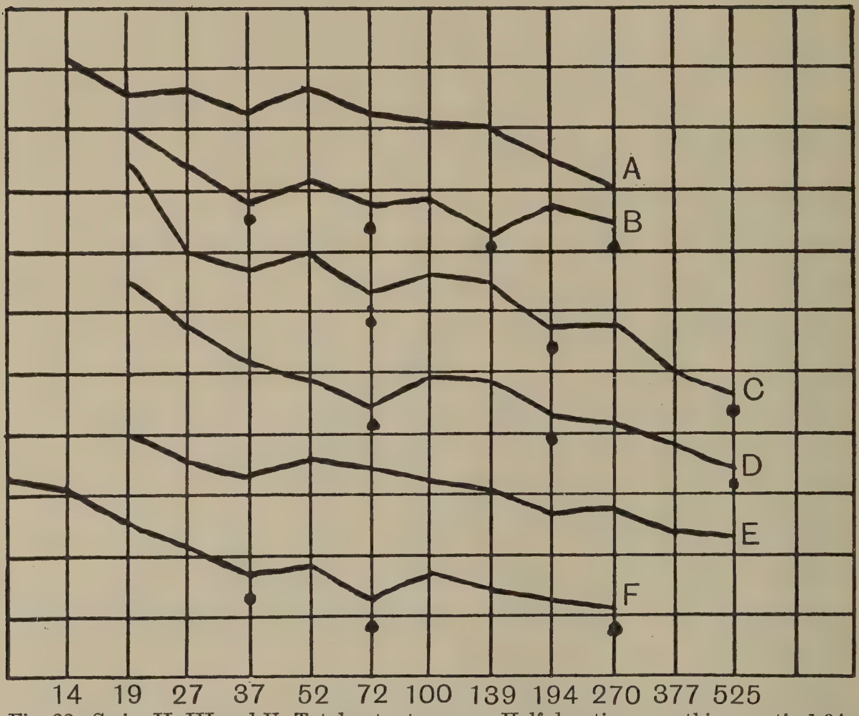

Fig. 22. Series II, III and V: Total output curves. Half-duration smoothings, ratio 1·94. Curve *A*, Series II; *B*, Series III; *C*, *D*, *E*, Series V (Full, Extracted, Straight); *F*, Total curve, Series II to V. For corresponding curve of Series IV (included in curve *F*) see Fig. 31 *H*.

(iii) If there are general and specific waves existing side by side, individual Extracted curves (of Addition, of Subtraction, etc.), will be characteristic in outline, but different from one another. Individual Straight curves should also be characteristic of the task, but rather less so, differing from corresponding Extracted curves in respect of any general waves they may contain. Full curves should be all alike, giving each a picture of the complex set up by the waves assumed to appear universally in every work curve.

(iv) If common elements are present as well as specific and general ones, Extracted curves should resemble one another somewhat by pairs or other small groupings. Individual Full curves should at the same time tend to lose their resemblance to one another, since individually they will contain waves common to given groups of tasks.

(v) If waves are neither general nor specific, all being common to two or more tasks but not to all, then the grand total Full, Extracted and Straight curves should tend to resemble one another. Individual Full, Extracted and Straight curves should, however, differ, and be constant in their differences. (The last point of constancy serves to distinguish this possible case from the next.)

(vi) Finally, there is the conception of a pool of general purposes wave-lengths, emergence of any particular wave being determined by factors other than those of the task, the subject, and so on, factors working in such a way that emergence seems to be chance determined, given waves coming sometimes in an *A* task, sometimes in a *B* task, in both, or in neither. In such a case, grand total Full, Extracted and Straight curves will come to resemble one another if based upon a sufficiently large number of records, since it will only be a matter of time for all the elementary waves to appear whether on the Full or the Extracted side. In similar fashion, individual Full, Extracted and Straight curves will come to resemble one another if enough records are taken. There will be no differences anywhere, save those of chance sampling among the elementary waves concerned.

Going through the various possibilities, it seems clear that we are confined to this last one of the general pool. There are no signs of constancy in curves where specificity or generality might be expected to hold; definite signs of constancy in respect of grand total Full and Extracted curves where mere weight of material is the most obvious reason for its occurrence. The next step, therefore, is to attempt a measure of similarity, to see whether, generally speaking, curves do come to resemble one another more and more as the number of records per curve is increased.

(*c*) *Average shapes.*

We here attempt a measure of similarity in respect of wave-length and phase. If a curve showing a perfectly simple wave form is cut into successive sections of one period length each, it follows that all the pieces so obtained will be similar in outline, each covering a complete crest-trough unit. If these sections are added together, an

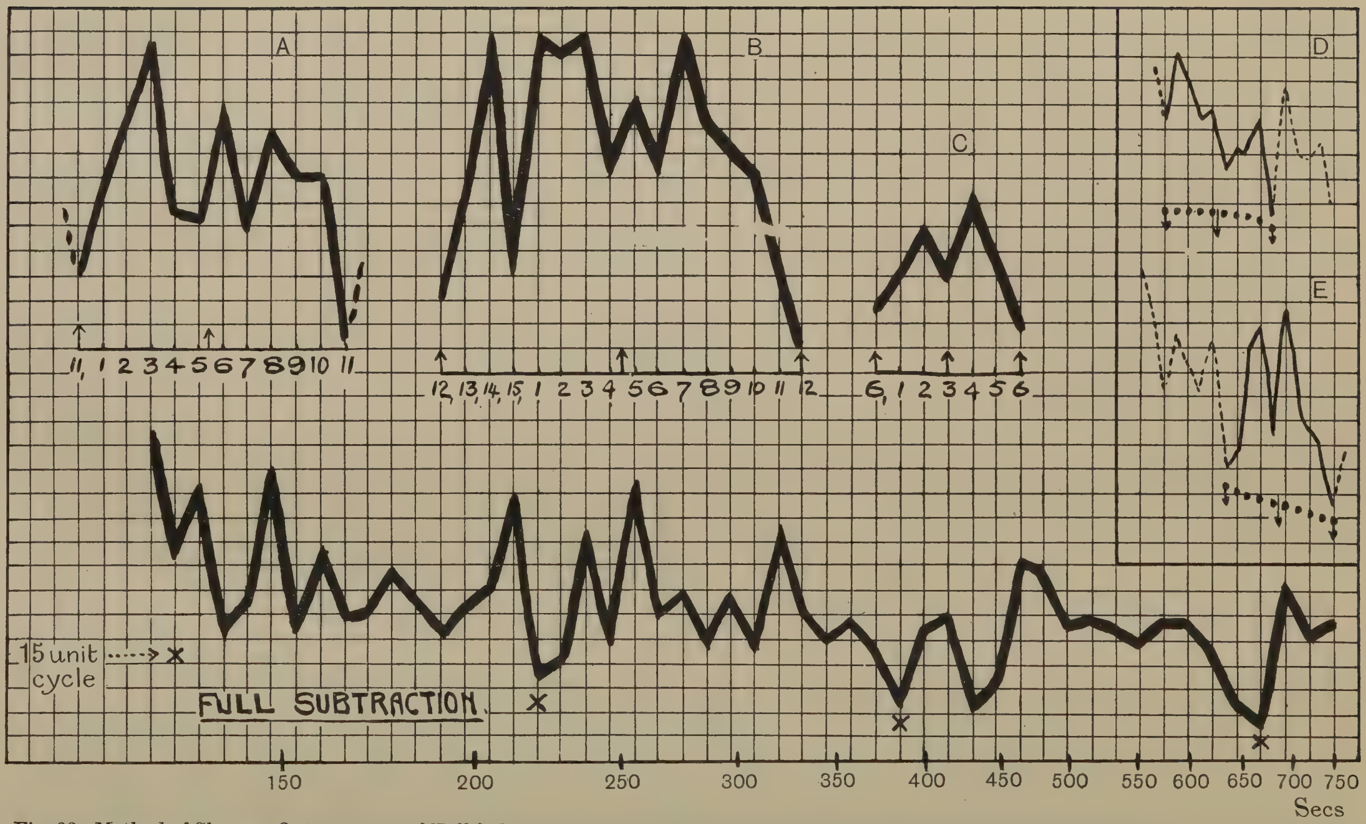

Fig. 23. Method of Shapes. Output curve of Full Subtraction, smoothed in units of 0·0160, and plotted against the log of time. Curves *A*, *B*, *C*, Typical "shapes," on bases of 11, 12, and 6 units respectively. Curves *D* and *E*, Nine-unit shapes (from one of the minor curves met with) the one of three layers, the other of four, illustrating the coming of a half-duration shift with the extra layer.

average shape or section is obtained. Such "shapes" afford (*a*) an expression of the average outline of the given wave, (*b*) an outline freed more or less from accidental irregularities, (*c*) a measure of the best-fitting trough sequence (for the lowest point in a shape is determined thereby), and (*d*) a measure of phase.

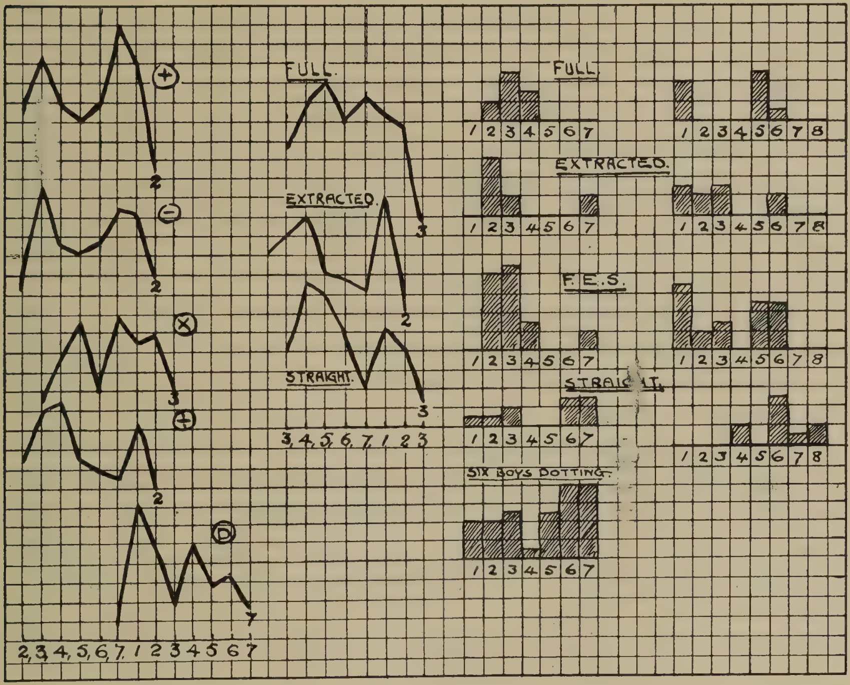

Fig. 24. Method of Shapes. Five shapes on left are of Extracted Addition, Extracted Subtraction, etc., on bases of 7 units of 0·0160. The three in the centre are of total curves. (The Extracted curve here is the sum of the first five.)

Things are not quite as simple with work curves. There are many waves present, and shapes attempting to measure single cycles will be determined not merely by the cycle nominally present, but also by all the others. In one important respect this is an advantage, since certain general characteristics of such shapes can be arrived at by arguments to be elaborated in a moment. A disadvantage is that shapes from different curves are not comparable, unless from corresponding portions of curve. Otherwise, of course, interference effects will not be the same in the two cases.

Curves to be treated were first expressed as smoothings with log units of 0·0160, 0·0020 or some such value. Such a curve (units of 0·0160) is shown in Fig. 23, together with shapes derived from it on bases of 11, 15 and 6 units respectively. For identification purposes, all shapes in units of 0·0160 (such as those here figured) have been given a number. Those labelled "1" are such that their best-fitting trough sequences pass through the log interval centred on the 445th second, any labelled "2" passing through the next succeeding interval, and so on[*]. This value was arbitrarily chosen. It happened to be characteristic of the first few curves handled[†].

Similarities between shapes could show in outline or in end-point placings. Taking first the question of end-points, it follows that if our fundamental assumptions with regard to constancy of phase and wave-length are correct, curves containing the same L.C.M. cycle should give shapes with end-points in corresponding places. And since, by hypothesis, weak curves vary more in respect of their L.C.M. cycles than do strong ones, we have to seek signs of agreement in shapes from statistically strong curves; of variability in shapes from weak curves[‡].

The shapes of Fig. 24 are on 7-unit bases[§]. They are of the five Extracted curves, but can be taken as typical of Full and Extracted work as well. Save for the fact that the Dotting shape manifests signs

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[*] The conventions of a shape are illustrated symbolically below. Let successive points in the curve be represented by letters (the best fitting trough sequence being in small letters).

1	2	3	4	1′	2′	3′	4′	1″	2″
A	b	C	D	E	f	G	H		
E	f	G	H	I	j	K	L		
I	j	K	L	M	n	O	P		
	w	X	Y	Z	w′				

To make a shape of four units base, the figures are set out as indicated (using, of course, as many "layers" as there are available) and summed. The points wXYZw′ when plotted form the shape.

Figures at the top indicate the columns in which the value at the 445th second actually falls (or would fall if the shape were continued), and so for the value next following it, etc. Such figures are also of the conventional method of marking the base of a shape.

[†] There is here a throwing back to an early attempt to establish a common trough point. It failed to produce any results of value because we were assuming 0·0160 as a unit.

[‡] We are really inverting the natural order of things. The general hypothesis was in very large part constructed to cover characteristics of the shapes now under discussion.

[§] Examples used here were mainly of the fifteen contributory curves of Series V, five each of Full, Extracted and Straight results. They are designated as of Full (Extracted or Straight), Addition, Subtraction, Multiplication, Division, or Dotting. In the few diagrams given, arithmetical signs or the letter D (dotting) indicate the nature of the task.

of what we shall call half-duration shifting, there is a strong tendency to agreement, end-points clustering about points 2 and 3 on the base-line scale.

Such tendencies to clustering may be expressed in the form of frequency curves. Two sets of such curves are given, one for certain groups of 7-unit shapes, the other for corresponding 8-unit shapes. We thus present the essential facts without having to draw the various shapes themselves.

Taking first the 7-unit distributions, that for Extracted shapes expresses the results already mentioned. The corresponding Full curve shows that shapes there cluster about points 2, 3 and 4. It follows, therefore, that Full and Extracted curves tend to agree not only in end-point placings, but also in the relative amount of variability present.

Straight shapes were of weaker material, being based upon 9–12 records apiece, as compared with 40–50 in the case of Full and Extracted curves. They are correspondingly more variable, covering almost the whole range of positions.

To show what happens with very weak material, figures for eighteen contributory curves from the Six Boys Dotting results were calculated*. They scatter over the whole of the possible range, showing, however, a strong tendency to bimodality, there being a clustering about positions 2–3 and 6–7.

Distributions for 8-unit shapes are of groupings of curves corresponding with those on the 7-unit side, save for the fact that the Six Boys Dotting material has not yet been treated on this basis. They very clearly illustrate the tendency to bimodality.

Scattering can be measured in terms of standard deviations. For this purpose a "strong" distribution (F.E.S.) was built up out of the individual Full and Extracted results together with the shape obtained by summing the Straight curves, making eleven shapes in all. A "medium" distribution was obtained by taking the individual Straight shapes together with those of the six boys individually. Finally, we utilised the eighteen original shapes of the six boys to form a "weak" distribution.

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* The six boys each attempted tests with four different kinds of squared paper, making 24 contributory curves in all. Of these we had, at the moment, 18 in terms of 0·0160 smoothings. It seemed hardly worth while to trouble about the other six cases.

For purposes of a comparison to be made in a moment, the results were pooled per boy, making six contributory curves.

Standard deviations were as follows:

Strong shapes	(F.E.S.)	40–50	records	apiece S.D.	0·28
Medium	,,	9–12	,,	,,	1·50
Weak	,,	1–4	,,	,,	1·70

In studying these figures it is necessary to keep two things in mind. If the curves are of random zigzags, and not of periodicities at all, then (*a*) weight in number of records per shape will be immaterial, whereas (*b*) weight in respect of number of shapes per distribution will mean that the S.D. approaches a fixed maximal value*.

On the other hand, if our general hypothesis is valid, if curves tend to become more and more alike with weight, then (*a*) weight in number of shapes per distribution will not matter (save, of course, that we must have enough to make the distribution and its S.D. stable), whereas (*b*) weight in respect of number of records per shape will lead to convergence on a minimal standard deviation. In the perfect case, when all curves are similar, this minimal value will be zero.

Taking the results as a whole, it is clear that we have been approaching the minimal value demanded by agreement in cycle lengths and phase, rather than the maximum demanded by chance. The rate of convergence too is fairly rapid. Using the ordinary rules of sampling, the observed difference between the S.D.'s 0·28 and 1·70 is at least six times its standard error.

(*d*) *Structure of shapes.*

By the conception put forward when drawing up tables of trough sequences, cycles must stand in relationship to one another. Those of 12, 24, 36, ... units start from a common trough at the point T_0, thereafter coming to common troughs after the lapse of L.C.M. intervals. So that, if a shape of 12 units base is attempted, its end-points must be determined not merely by waves of the cycle upon which it is nominally based but also by those that are multiples thereof. That is to say, troughs of a 24-unit cycle will fall sometimes at one end of a

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* By chance, increase in the number of records per shape will merely mean that it tends to become flat and indeterminate. If there are any "end-points" at all, they will occur (in the long run) equally often at all the possible points along the base line. The distribution will be flat, and the more shapes we have, the flatter it will be. With a perfectly flat distribution on a base of 7 units the S.D. will be of 2·00, and this is a maximal value. (It cannot be exceeded, for if it were, the distribution would in effect be piling up at the ends of the base line. But since points 7 and 1 on the base of a shape are really adjacent, such a piling up at the apparent ends would really mean a concentration on adjacent points, i.e. a reduction in the S.D. and not an increase.)

12-unit shape, sometimes at the other. The effect, therefore, of a cycle of twice the nominal length is seen in an alternate raising and lowering of the ends of shape, equilibrium coming, periodically, whenever complete cycles of the longer wave have been added in. Similar arguments hold for cycles that are three, four, or more times the nominal one, save that, in the long run, their influence upon the depth of the resulting end-points of the shape becomes smaller and smaller, the longer the said alien cycles happen to be.

A cycle of, say, 18 units will have its troughs falling sometimes at either end, sometimes in the middle of a 12-unit shape. A double hump will be produced. If the longer cycle is $2\frac{1}{2}$ times the nominal one, the same kind of thing will happen, and so on. The longer the alien cycle, however, the less the degree of double humping produced.

In similar fashion, treble and quadruple humpings will be produced. With these, however, it should be noted that one assumes a large number of sections to be available. With a small number of sections the resulting shape may seem to be one-sided, being deeply cut in the centre and at perhaps one of the quarter points, but not at the other (cf. shapes of Fig. 23).

Shapes cleft in the centre should occur most frequently. Or stated more accurately perhaps, centre clefts should be deepest. For let a shape be acted upon by cycles that are $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, ... of its length. Such alien cycles will tend to cut the shape into halves, quarters, and so on, but clearly they will come together most frequently at a point represented by the fractions $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \dots$, i.e. at the centre of the shape. A threefold tendency should be less frequent, the requisite fractional values being fewer, and so on. This kind of thing actually happens. Even shapes on bases of 7 units are more often than not split into portions of 3 and 4 units width. There is rough centre cleavage.

It might of course be urged that, in this particular case, the phenomenon is of chance, the number of possible arrangements on a 7-unit base being limited. Reference may therefore be made to the shapes of Fig. 25 which are in terms of 55 units of 0·0020. (The base is thus approximately as for 7 units of 0·0160, but with many more points in it.) With as many units as this, there is room for an enormous number of possible outlines. Yet the simplest and perhaps the only way of describing the results is to say that outlines are of double, treble, or quadruple humpings.

If a shape is acted upon by alien cycles that sometimes raise one or other of its nominal end-points, while at the same time giving a

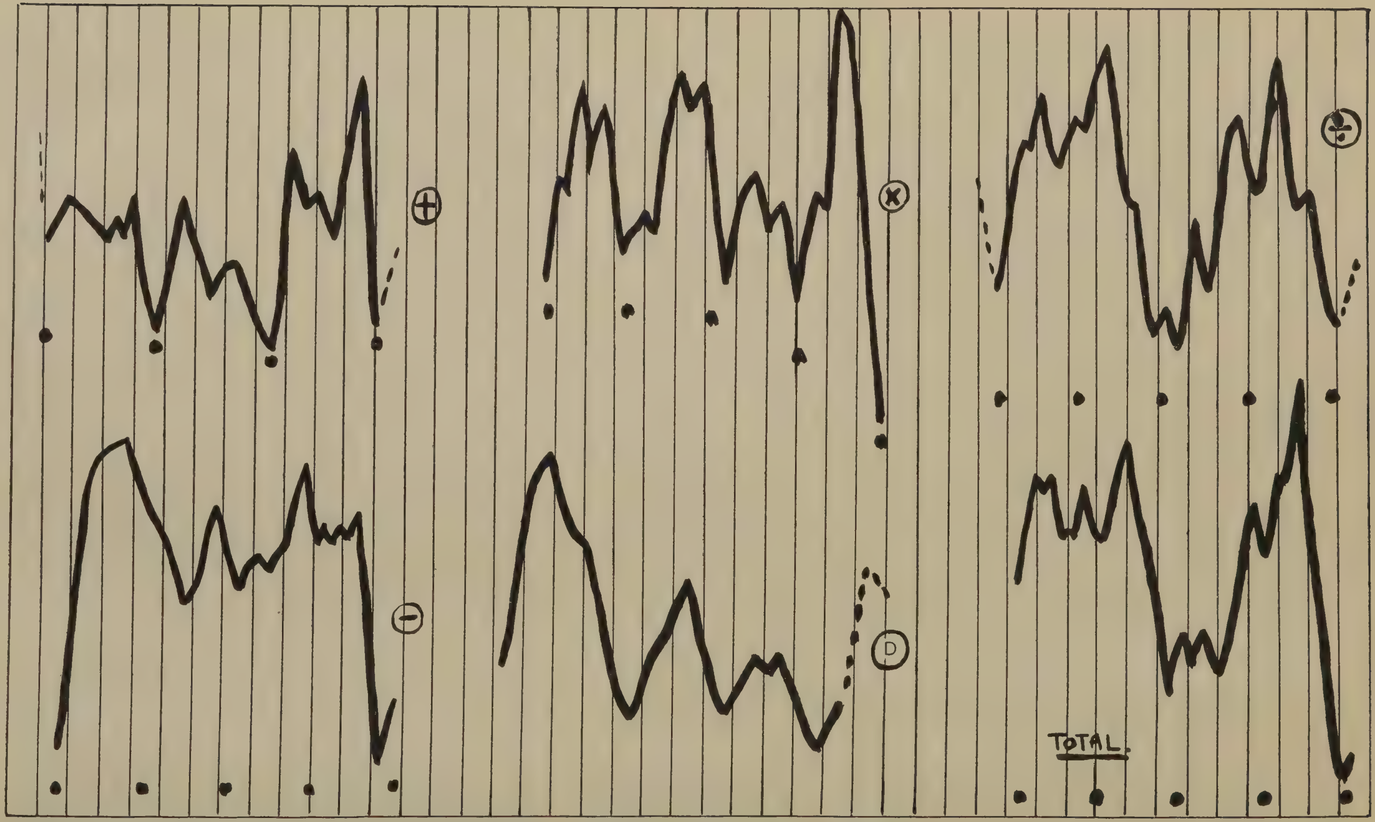

Fig. 25. Method of Shapes. The five contributory Extracted curves, with their sum, expressed as shapes on bases of 55 units of 0·0020. They are of the same material, and on approximately the same bases as those of Fig. 24.
(7 × 0·0160 = 0·1120; 55 × 0·0020 = 0·1100.)

strong tendency to centre cleaving, then, on more and more layers or sections being added, half-duration shifts may occur, the nominal centre trough becoming dominant for the time being. There is here a possible explanation of shapes such as those of Fig. 23 *D* and *E*, where the addition of an extra layer has actually shifted the end-points through a half duration.

A shape of given length could well be set up by related cycles, even if the nominal one is not well enough marked to dominate the curve. An alien cycle of, say, 18 units might in itself generate a shape of 12 units length. Since cycles of 12 and 18 units must have troughs in common as often as possible, it follows that a shape generated in this way could easily have its end-points falling as for a true cycle of the nominal length. It is in this way that one would explain part of the tendency for curves to conform in respect of end-point placings. For it cannot be assumed in every case that curves giving shapes of corresponding end-points must also show a definite L.C.M. cycle of that given length. Their cycles may only be related in this present sense.

If a shape of given base is actually determined by alien cycles, and if its end-points fail to fall as they would were the nominal cycle present, it follows from what we have just said, that the apparent discrepancy must be some simple fraction of the nominal cycle length. Generally speaking, therefore, deviations of end-points from the mean in frequency diagrams such as those of Fig. 24 are not necessarily of continuously variable amounts or "errors." By hypothesis, they are more likely to be of jumps of one-half, one-third, or some other harmonic fraction of the given duration.

The tendency to bimodality in those said frequency diagrams, a manifestation otherwise inexplicable, seems to be adequately covered by considerations of this kind. By hypothesis, half-duration shiftings are more likely than others*.

(*e*) *Individual differences.*

The experiments were not framed for the purpose of making individual differences manifest, but what signs there are may be read. No child attempted more than two tasks. Of those taking Addition as one of their performances, a quarter took Subtraction, another quarter Multiplication, and so on. This means that Full and Extracted curves of the same task name are by the same children, but that there is only an overlap of a quarter of the subjects when comparing with curves of other names, whether Full or Extracted. Individual differences should therefore show, if at all, in that direction. From the results, of course, it is fairly clear that differences are equally scattered in all directions.

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* The above arguments are expressed generally. In studying particular cases, attention is frequently taken by manifestations of hybrid cycles. Thus e.g. shapes from curves apparently opposed in phase are obviously likely to show half-duration shifting.

Individual Straight curves are by some of the children concerned in Full and Extracted curves of the same designation. They should therefore vary among themselves, but resemble Full and Extracted curves of the same name. Again, variations are not in this sense. They seem to be entirely of weight of material, rather than of any other factor.

(*f*) *Of geometricity and phase in Extracted curves.*

Let it be supposed that apparently geometric waves of ratio 1·94 are set up at the commencement of a sandwiched experiment, and that when at the 60th second change is made from the *A* to the *B* task, the said geometric waves carry straight on, no others being generated. It follows that troughs appearing at 64, 124, 240, ... seconds in the Full record will come at 4, 64, 180, ... seconds in the Extracted one. The sequence is neither geometric, nor does it conform to the given phase constants of a ratio 1·94 cycle.

Yet if our results mean anything at all, it is that Extracted curves conform as closely to the geometric system, whether in wave-length or phase, as do Full or Straight ones. Their "shapes" have end-points in corresponding places, and so on. It is inconceivable that we should have arrived at such a result were the curves anything but what we have claimed them to be, geometrically determined, with waves beginning at the point of change.

4. CONCLUSIONS

When two tasks follow one another without a break, it seems that waves may cross over from one task to the other, thus being common, while other waves are as definitely generated at the point of change. Whether any are there extinguished, we cannot say.

There is an increase in output at the point of change, or within a few seconds after it. There seems to be tapping of new sources of energy, followed, of course, by a fall off. Whether the "decrement" thus manifested is associated with the waves as such, each, as it were, carrying its own decrement, is undecided.

Individually variable, Full and Extracted ("general" and "specific") curves come to resemble one another as more and more records are brought into account. It is therefore inferred that given wave-lengths will in the long run appear as often on the Full side as on the Extracted. They are not specific to definite tasks, nor are they general in the sense that they must always appear. They may, however, be specific to psychophysical mechanisms of a general purposes nature, such that they can as readily appear in work on one particular task as in any other.

Individual differences between the subjects must be small compared with those due to sampling of wave-lengths. All traceable differences are of weight of material alone.

Incidentally, the results give evidence in favour of the general hypothesis. With regard to shapes, the findings are (*a*) that their outlines tend to be harmonically cleft, centre cleavages being most common, (*b*) frequency curves showing the distribution of end-points in a group of shapes are often bimodal, and (*c*) by varying the number of layers in a shape, end-points may often be shifted through a half duration, or other harmonic fraction of the nominal duration concerned. These findings we interpret to mean that they are based on L.C.M. cycles that are of comparable lengths and constant in phase, as assumed during the discussion of inter-cycle relationships in Part I. With regard to Extracted curves, the findings are that they behave in precisely the same way as do Full and Straight curves. This they could only do if their waves are geometric and if they are constant in phase when referred to the point of change-over as their true zero on the scale of seconds of time.

C. PERIODOGRAMS AND SERIAL CORRELATIONS

1. The problem.
2. Methods: periodogram analysis, serial correlations.
3. Results and findings.
    - (*a*) Curves chosen for discussion.
    - (*b*) Of rhythm in the results.
4. Conclusions.

1. THE PROBLEM

Standard methods of analysis, such as that of the periodogram, are not really applicable to curves as short as those with which we deal. Reliability cannot be attained with less than about twenty periods of the waves concerned. To get twenty periods of ratio 1·94 is not humanly possible—unless we break into the first 5 seconds of work. We have not been able to do so. Keeping outside the first half-minute, as we have done, it follows that our subjects would have had to work for months on end without food or sleep to make the tale of periods complete. They are geometric.

Again, such statistical methods are usually applied to curves in which but one wave is suspected, or at most a few. We are, by hypothesis, dealing with a large pool of waves, and complications are to be expected. That being so, we have applied the two methods of analysis not because any one result could possibly be significant, but in the hope that the results as a whole might possibly give indications analogous to those of, say, the shapes of the preceding section.

2. METHODS

If a shape, as defined in Section B above, is based upon a true period, then its outline will be of maximum amplitude. Shapes not based upon true periods will be damped. In practice, shapes have to be calculated for all the possible wave-lengths within a given range. Measures of their respective amplitudes are then plotted against a scale of wave-lengths, the resulting "periodogram" indicating which wave-lengths are more probable than others. Strictly speaking, measures of amplitude should be in terms of Fourier Coefficients, or perhaps correlation ratios. A simple method, only to be used for rough surveys, is to measure the vertical distance between the highest and lowest points in a shape, dividing by the number of layers or sections concerned. Only results of this simple kind were used during the research, greater accuracy being sought later by the method of Serial Correlations.

Following the example of others who have used the method, our first curves were cleared by expressing them as deviations from a running average*. Such smoothings were later abandoned, return being made to the original work figures. To begin with, if the running average is by steps of, say, 55 seconds, waves of that size or less will be relatively unaffected. On the other hand, any of, say, 300 seconds period will be automatically flattened out. Or with geometric waves, the long crest-trough units at the end of a curve will suffer as compared with shorter ones of the same wave form nearer the origin. One might, of course, use geometric intervals for purposes of averaging, but even so the same objection would hold. Waves of large ratios would be damped as compared with those of small.

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* See Appendix C.

For comparative purposes, periodograms should be calculated from corresponding portions of the various curves. (This same problem was mentioned with reference to shapes, cf. p. 79.) Those of Fig. 27 were therefore based on curves cut to a uniform length of 300 seconds. Those of Fig. 26, however, are on the full length of the curves concerned.

Periodograms of Fig. 27 are also on material smoothed by expressing the figures as deviations from a running average of 55 seconds. Those of Fig. 26 are from unsmoothed figures. Whether smoothed or not, the original work-curve figures were next expressed in terms of logarithmic time units. Those used varied in size from 0·0160 to 0·0010. One of 0·0020 proved most convenient, all work with larger units being comparatively damped*. Periodograms as calculated by the simple method will be found to slope to the left, owing to increase of damping with the number of layers in the shapes. (There are more layers or sections with shapes of short wave-lengths than with long.) Such damping comes from (*a*) averaging of chance factors, and (*b*) smoothing out of cycles alien to the one actually measured. If presented uncorrected for this slope†, periodograms have gaps in their outline to indicate where transitions took place from two to three, or from three to four-layer shapes, etc.

If two copies of the same curve are correlated together, the resulting coefficient is of course $r = 1{\cdot}00$. If a wave is present, and if one copy is shifted through exactly one period relative to the other, the coefficient will still be $r = 1{\cdot}00$. If the shift is of half a period, $r = -1{\cdot}00$, and so on. Correlating in this way for varying degrees of shift, and plotting the results as in a periodogram, the curve of Serial Correlations is obtained. For some of our S.C. curves, the "r" value has not been fully evaluated. Corresponding "*xy*" values have been given instead. For our purposes, they are equally informative. Some of the S.C. curves will be found to slope to the right. This is usually due to the fact that we have taken "*xy*" values.

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* Much time was spent dealing with units afterwards proving to be unsuitable. Incidentally, it might have been better to use units of 0·0016. The problem was considered, but it would have meant more work, and as one was not quite sure of the general hypothesis at that moment, the simpler units were employed.

† Correction was made by multiplying all values by $\sqrt{N}$, N being the number of layers concerned.

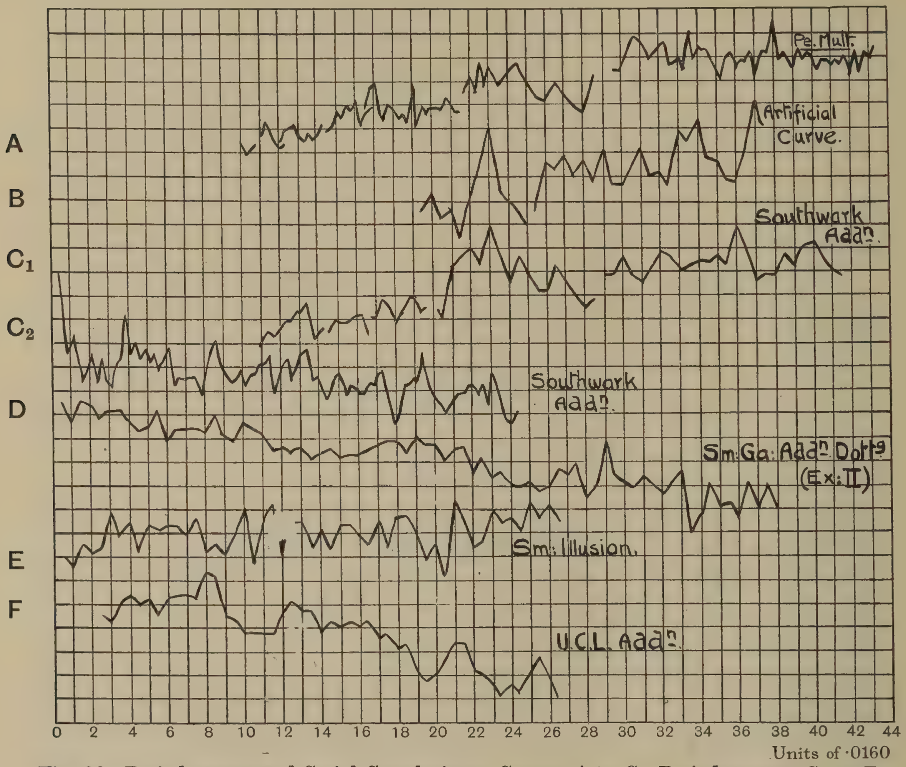

Fig. 26. Periodograms and Serial Correlations. Curves *A* to C_1, Periodograms; C_2 to *F*; Serial Correlation curves. There is no significance in the fact that the periodograms were not continued back to cover wave-lengths near the origin. C_1 and C_2 are based on the same material, thus enabling comparisons to be made between the two methods of calculation.

3. RESULTS AND FINDINGS

(*a*) *Curves chosen for discussion.*

Two aspects of the work are illustrated in Figs. 26 and 27. In the first are typical periodograms of the simple kind mentioned above, together with a few S.C. curves. In the main they are of our long curves. Fig. 27 gives periodograms of the grand-total curves of the resear

There is also shown the sum of their periodograms, and the periodogram of their sum*.

Examples of periodograms as usually given in text-books are of much simpler cases than those dealt with here. When, for example, there is but one wave in the original material, the periodogram will tend to have but one strongly upstanding crest in its outline, and so on. But with our material, where many waves are postulated, they may fail to be made manifest simply because they are so numerous. Every point in the periodogram may, as it were, be a crest, no room being left for intervening troughs.

Curve *B* of Fig. 26 is therefore given as a standard of comparison. It is from an "artificial work curve" constructed by adding six waves of known period and amplitude†. As drawn, 90 units of this curve have been made to correspond with 18 of the work curve scale, i.e. the crest nominally at the 23rd unit in the artificial material is really of 5 × 23 natural units.

The point to be noted is that major and minor crests are by no means strongly marked relative to one another. In fact, most of the ups and downs might be mistaken for accidental irregularities, although by construction there can be none present. Comparing with the experimental results, it may therefore be ruled at once that work-curve periodograms are no more irregular than this example of known origin; that their apparent major crests seem to be as strongly marked.

Before leaving the example, it should be noted that it raises two problems. One is of prime-numbered wave-lengths. The other is of empirical likeness with the curve of Southwark Addition, a likeness investigated at some length early in the research by means of shapes, etc. These early results are not presented, the periodograms showing the relationship sufficiently well. That of Southwark has crests at 36, 23, 19, 17 and 13 on the given scale‡. The "artificial" periodogram has large crests at 37 and 23 respectively, while from the known constitution of the original curve, crests may also be inferred to occur at 19, 17, 13, 11 and 7 units. It is no wonder resemblances were noticed between the original curves*.

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* If two curves contain the same waves, their periodograms will of course be identical. And that without any regard to phase. Obviously, however, the periodogram of their sum will suffer if there are phase differences. The curve may be so flat as to be useless for purposes of analysis.

† Periods were of 7, 11, 13, 17, 19 and 23 units. Amplitudes were constant throughout, and all waves were made to begin from a trough from zero time. Prime-numbered values were chosen to ensure that no simple repeat (or L.C.M.) cycles could appear. Curiously enough, the resulting curve seemed to have a complex cycle of 90 units length, despite the apparent bar on short repeat cycles. The said 90 unit cycle was found to die out when more and more of the curve was constructed, as of course it might be expected to do. See footnote, p. 92.

‡ Curves C_1 and C_2 must be taken together. They cover the two ends of the curve.

(*b*) *Of rhythm in the results.*

When using the S.C. method, the effect of shifting one copy of the curve through a complete period relative to the other is to bring the correlation back to $r = 1{\cdot}00$. The same thing happens with shifts of 3, 4, 5, ... times the period. In other words, S.C. curves (and analogously periodograms) should be periodic in terms of the original periods.

Examine now Fig. 27 *G*. It seems to be rhythmical, having crests at approximately 11, 15, 19 and 21 units, with possible smaller ones at 13 and 17. But the curious fact emerges that the sequence is entirely of odd-numbered values, some of them primes. Now if the original curve is periodic, one would have expected at least a few even-numbered values. In fact the chances are against odd-numbered ones at all—provided the unit of measurement is suitable†—and we may pertinently enquire how Fig. 27 *G* arose at all. Why is it odd-numbered, why the primes?

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* It was with this resemblance that measures of the unit of wave-length began, some long time before periodograms were attempted (or even the analysis of Table III), when one was yet attempting to fathom the structure of "shapes," such as those dealt with in an earlier section. It was noted that the shapes of Southwark Addition and of the artificial curve are obviously similar in outline, 90-unit shapes on the artificial side going with 18-unit shapes on the work curve side, and so on, *pro rata.*

Now if 90 natural units go with 18 of the 0·0160 scale, it follows that the true work curve unit of periodicity is of 0·0032 (for Table III, of course, it is assumed to be of 0·0016). The fact that a unit was obtained in this way by comparison with a set of prime numbered wave-lengths was very puzzling. It could not be supposed that the Southwark curve—or any other for that matter—is likely to be of prime-numbered cycles alone.

The matter was explained by showing that cycles of 90 units length can be set up quite commonly by other combinations of waves (as later we argued when drawing up Table III), and it seemed clear that the addition of such other cycles to the present artificial curve would merely result in the accentuation of the apparent 90-unit cycle. Whence, of course, came the argument that similarity between the curves in question is due to the fact that both are based on waves that are whole-number multiples of a unit, and not necessarily to the fact that the artificial curve happens to be of prime-numbered wave-lengths.

This does not explain why work-curve material may seem, on occasion, to be prime numbered. For that aspect of the problem see further remarks on grand total periodograms in the text.

† If an elementary wave of 3 units is present, there should be crests in the S.C. curve at 3, 6, 9, 12, 15, ... units along the base line. Half these values are even. If the elementary wave-length is even numbered, then all the resulting values must be even.

Light is thrown on the matter by the control periodogram, Fig. 26*B*. It has crests at 23 and 37 units respectively, odd numbers, and primes to boot. Yet we know that its tiny elementary waves must have quadrupled themselves at least before reaching the size of cycles of this order. In this particular case, the coming of primes is obviously due to the use of intervals coarser than those of the original curve*. The same kind of thing may have been happening in Fig. 27*G*.

Rhythms in curve *F* of Fig. 27 are more complex. Starting from an incomplete crest at $3\frac{1}{2}$ units†, and working forward, it looks as though the curve may be moving by triple groups of crests, one such triad coming at points $3\frac{1}{2}$, $5\frac{1}{2}$, $7\frac{1}{2}$, another at $13\frac{1}{2}$, $15\frac{1}{2}$, $17\frac{1}{2}$. Both groups are thus of the same width. Moreover, their width is equal to that of the intervening space. Speaking of "triads" and "spaces" to avoid circumlocutions, it follows that if this system were continued, spaces would be centred at $0\frac{1}{2}$, $10\frac{1}{2}$, $20\frac{1}{2}$, $30\frac{1}{2}$, ... and so on. The sequence has its origin nearly enough in zero.

The obvious thought at this point is to return to Fig. 13*A* and the conception of series among the natural numbers. In that diagram will also be found triads and spaces, both tending to be commensurable in size‡. Such a diagram however is not a periodogram. It is of factors in given numbers. On the other hand, if corresponding waves are present in a curve, its periodogram might look something like this present curve. There should be crests corresponding with all the highly factorisable wave-lengths, and the amplitude of those crests should be determined to some extent by the degree of factorisability. For the moment therefore we may regard Fig. 13*A* as akin to a periodogram.

The argument was taken a step further when it was realised that the curve has not suffered smoothing. The points were therefore pooled by fives to give curve *B*. The result is to increase the likeness to the empirical periodograms. Triads and spaces still appear. Moreover, there is some slight tendency for solitary crests to be pushed up in the middle of spaces, although it is true that they do not appear as markedly as in Fig. 27*F*. While if we go on to complete the illusion by dividing all abscissal values in Fig. 13*B* by ten*, a resemblance to Fig. 27*G* appears as well, crests coming at points 7, 11, 17 and (19) on the new scale.

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* In all calculations from the artificial curves, its original figures were pooled by fives. it being desired to introduce an element of smoothing analogous to that suffered by work curves when assessed in terms of output per 5 seconds, when further they are geometrically smoothed, and so on. The wisdom of introducing this complication is illustrated by the present considerations.

† Crests at this end are relatively small. This is a damping effect, due to increase in the number of layers or sections, and allowance should be made for it when comparing with larger crests at the other end of the curve.

‡ Triads and spaces do not form a perfect series, double spaces occasionally appearing, We have assumed the periodogram (Fig. 27*F*) to be of a perfect series, but it is obviously too short to enable us to say whether it too has double spaces, etc.

There is thus not only an analogy with the triads and spaces of the one periodogram, but also with the dominantly odd (and prime) numbered values of the other. It has to be admitted that the argument is by analogy. The case is not necessarily proved in the strict sense. To go further along these lines we should need to show just how closely Figs. 13*A* and *B* agree with corresponding periodograms, enquiring also into the effect of varying degrees of smoothing†.

4. CONCLUSIONS

The Methods of Serial Correlations and of Periodogram Analysis are not strictly applicable to geometric periodicities of the kind met with. The curves are not long enough. A smoothing interval of about 0·0020 proved most suitable. For practical purposes, the S.C. method is probably as quick as the rough periodogram method, and the results are more likely to be reliable.

The periodogram for the grand-total curve of the research tends to have crests at odd-numbered multiples on the 0·0160 scale. To explain this emphasis upon odd and sometimes prime-numbered cycle lengths (where one would have expected highly factorisable values), it was shown that the same kind of thing happens when the curve of degree of factorisability in the natural numbers is smoothed, or

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* Just as units of 0·0016 can be said to be pooled to give those of 0·0160 in the periodograms.

† The triads of Fig. 13*B* are not strictly analogous with those of Fig. 27*F* in their position relative to zero. Continued back, the series of the latter curve begins (presumably) with a space centred at 0½ units. Fig. 13*B* has a triad of crests centred there instead. Both sequences are of course symmetrical about zero, as they should be if we have to do with periodicities, but they are not of the same kind.

We might have arrived at a closer degree of agreement by varying the degrees of smoothing, but the problem is too involved to deal with quickly. To begin with, the work curve units were of 0·0020, and not of 0·0016 as they might better have been. Add distortion introduced by reason of the use of 5-second intervals, experimental errors, etc., and it becomes clear that we are not easily going to get a hypothetical curve to match the empirical ones in all respects.

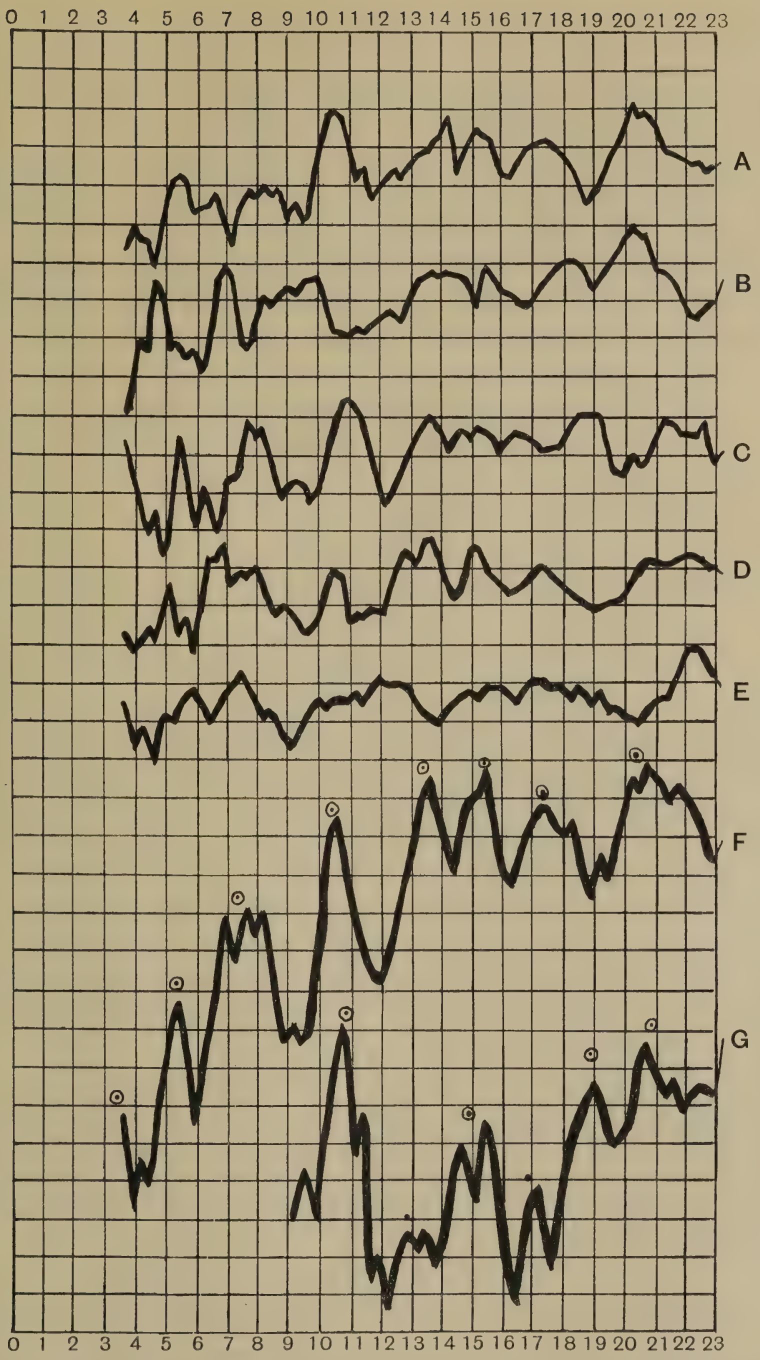

Fig. 27. Periodograms: Series II, III and V. *A*, Series II. *B*, Series III. *C*, *D*, *E*, Series V (Full, Extracted, Straight). *F*, Sum of periodograms *A* to *E*. *G*, Periodogram calculated from sum of the curves of Series II, III and V.

again when a curve of known and complex constitution is smoothed before being treated by the periodogram method. It is probably due to the use of coarse measures.

The sum of the periodograms of the major curves of the research also moves in a rhythm akin to that shown by the curve of factorisability among the natural numbers just mentioned.

On the whole, therefore, although there is nothing of statistical significance in the results, it may perhaps be said that they are not inconsistent with the general hypothesis.

D. POSSIBLE ARITHMETIC WAVES; IMPOSED RHYTHMS

1. The problem.
2. Data.
3. Possible interpretations.
4. Conclusions.

1. THE PROBLEM

There is room in a complex curve for waves of all kinds, whether geometric or not. Two or three of the early examples were actually indexed as showing waves of 50 or 60-second period. Fig. 16 seems to have a wave of that period. Another early example, that of Sm. Illusion (Fig. 9), was thought to be of that same period even before geometricity was noticed in it.

With the drawing up of grand-total curves (Figs. 20 and 21) a tendency to high output at the ends of minutes was noted. In the grand-total curve of all series (Fig. 28*A*) there was no mistaking it. Most obvious crests appear at 60, 120 and 240 seconds, with minor ones at 180 and 300 seconds.

The 60, 120, 240, ... sequence is geometric, and of ratio 2·00. But when account is made of the minor crests at 180 and 300 seconds, it can fairly be said that there are, as it were, waves of arithmetic periods parasitically disposed about the slopes of the familiar 45, 90, ... second sequence of crests, and the problem of their nature and origin arises.

2. DATA

The first attack was by way of S.C. curves, using arithmetic periods. Those of Fig. 29 are some of grand-total results, some of certain

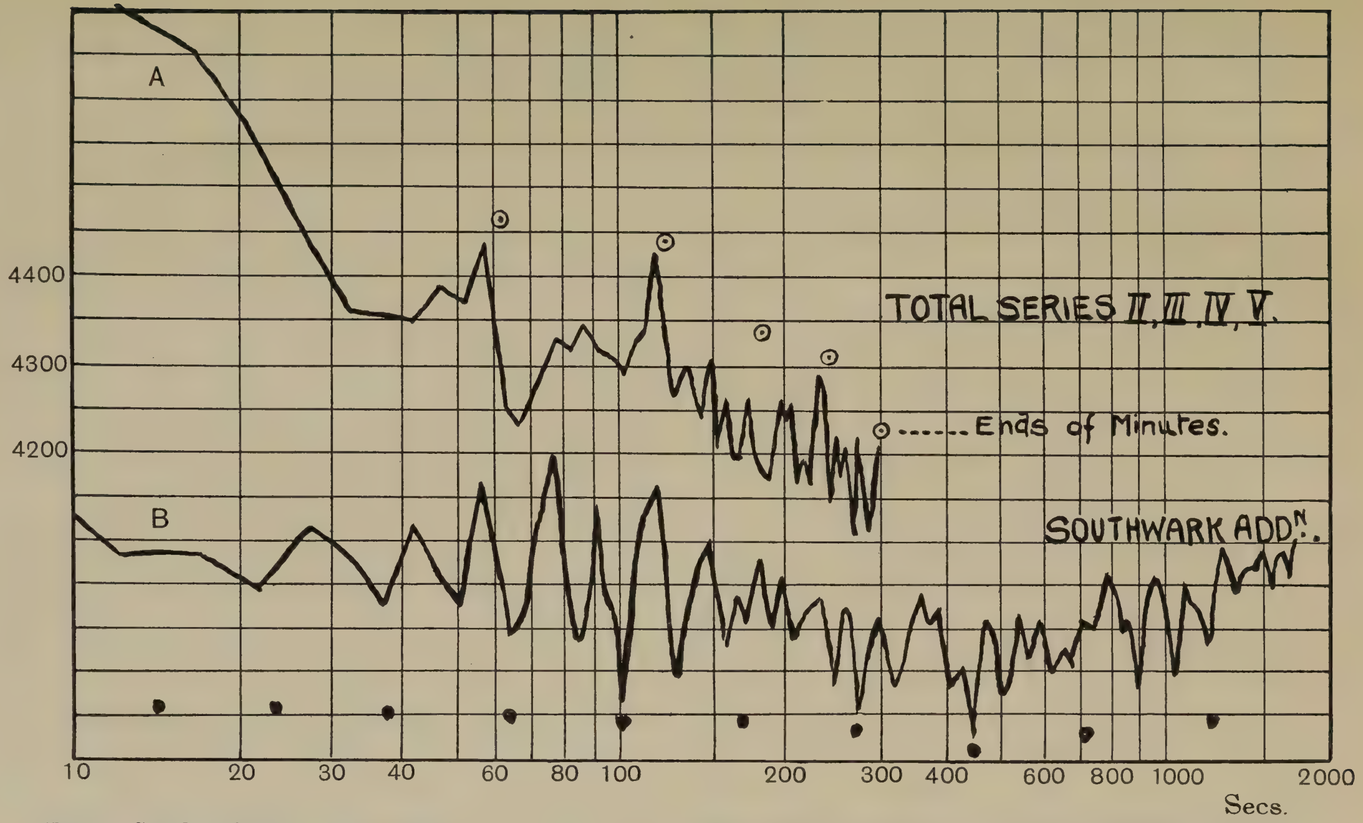

Fig. 28. Grand total output curve: Series II to V. The curve of Southwark Addition is included for comparison. Marks under the curve indicate lengths of approximately 2 × 66 log units.

subsidiary curves*. In every case, there are signs of rhythms of approximately 15, 20, 30 or 60 seconds, as the case may be. The U.C.L. curve was later continued, being calculated for as many periods again as those shown. The 30-second wave broke down almost immediately, showing that it is approximate only. The same thing happened with a periodogram for the Stepney Division material (not shown). It was of the U.C.L. type, so that in those cases at any rate, the apparent 30-second cycle is not exactly of that size. The evidence might alternatively mean that it does not extend much beyond the 300th second. One would be inclined to say that both findings are true.

The fact that the apparent cycle may be of exact minutes or a fraction thereof, led at once to an investigation into the possibility of imposed rhythms. Flugel (15) found in his experiments that the giving of double time signal at the end of every minute meant a loss at those points, his grand-total curve being more or less flat save for the said deep troughs at the ends of minutes. There were no double signals in our own work, but most of the signals had been given verbally, the experimenter taking the times from a stop watch. It is quite possible that he gave some subtle signs, say of relief, each time the hand of the watch completed a cycle.

It was at this stage that the U.C.L. curve was obtained, time signals being given by a bell controlled by a distant metronome. No one was in the room save the subjects themselves. The resulting S.C. curve is as rhythmical as any.

One way of dealing with an imposed rhythm is to set the subjects working at different times relative to one another, and the environmental factors that might conceivably be responsible. In effect, this is what happened with the *B* tasks of sandwiched work. Some of the subjects changed at the 60th second, some at the 65th, and so on. Extracted curves ought therefore to be distinctly complex or even meaningless when examined for signs of a rhythm imposed from outside. In any case, they should be less clear than corresponding Full curves in which a rhythm of this kind would have a better chance of showing itself. As it happens, the S.C. for Extracted work is as clear or even clearer than that for Full, having crests quite neatly spaced at intervals of 15 seconds.

There is still a possibility that signs of arithmeticity might arise owing to the use of time intervals at all. The subjects changed lines at the end of 5-second intervals. A corresponding rhythm may have been generated. If so, if there is a 5-second cycle in the curves, its simpler manifestations would elude us, crests and troughs being absorbed within the totals of work done per interval. But more complex cycles would be possible, subjects coming to emphasise, say, every second interval, or every third interval, according to one or other of the common metrical rhythms. Assuming this to be so, that two-three- or even manifold groupings of the 5-second intervals are

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* Total curves were first expressed as deviations from a running average of 55 seconds interval. Others were used unsmoothed. Figures are usually to the nearest 5 seconds.

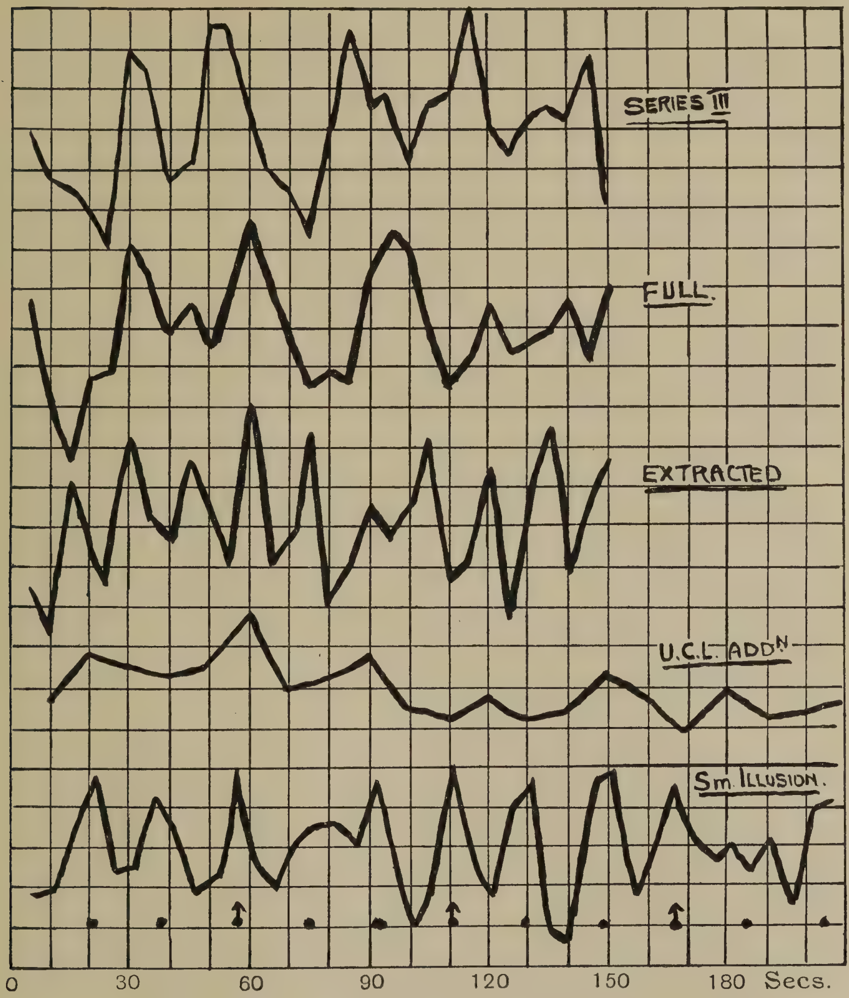

Fig. 29. Periodograms: arithmetic waves.

possible, the L.C.M. conception may again be invoked. If cycles of 5, 10, 15, 20, ... seconds are possible, then, on summation, complex cycles of, say, 6 or 12 times that unit are more likely to appear than not. These would be of 30 or 60 seconds, giving a conceivable explanation not only of the appearance of arithmetic cycles at all, but also of their apparent size.

As an argument against this notion there is the curve of Sm. Illusion. It is as arithmetic as any, although there were no time intervals at all. The manifestations are therefore not of the interval entirely, even if they may be partly responsible for it in cases where time intervals are used.

3. POSSIBLE INTERPRETATIONS

The first explanation coming to mind was on the lines just mentioned. Examining Fig. 28*A*, there actually seemed to be an end of the minute cycle, although it appears as clearly to be of minor importance. (The crests at 60, 120, ... seconds could be cut off without altering the essential nature of the curve.) Granted therefore that such curves are inherently geometric, weak arithmetic cycles might have arisen in some way during experimentation, possibly by reason of the time intervals. Being weak, they would not interfere with whole-duration smoothings to any great extent, and would usually become apparent only on summation, geometric waves being of different wave-lengths flattening themselves, the presumed arithmetic cycles being of the same or but few periods, building one another up.

But however true it may be for grand-total curves, this is not a full explanation. The curve of Sm. Illusion is of but one record. Arithmeticity has not here been accentuated on summation. Or again, Fig. 28*B* is of a different type from the *A* curve above it. It is of but six records (the argument of summation again not holding), yet its apparent arithmetic cycles are so strong that were they cut out, the curve would be destroyed*. The relationship between arithmetic and geometric waves in it is so intimate that the question arises whether there actually can be two kinds of wave at all. Is it that we are misreading the curve, reading arithmeticity into geometricity, or *vice versa*?

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* For the first 100 seconds it can be read in terms of an apparent cycle of 15 seconds. Thereafter, it becomes one of 30 seconds for a time, and finally of 60 seconds. The manifestations become weak outside the first 5 minutes, and seem to have died completely by the end of 10 minutes.

We here return to one of the key problems of the research. Taking the 60, 120, 240, ... second series of Fig. 28*A*, it seems to be geometric as it stands, but were the minor crest at 180 seconds magnified a little the series would at once become arithmetical. At first sight everything seems to depend upon this one value, and we have often found it as difficult to decide whether given series are of the one kind or the other. It was for this reason that whole and part-duration smoothings were made. For were the curves arithmetic in this present sense such smoothings would be impossible. The evidence from geometric smoothings is overwhelmingly against arithmeticity. But with the present S.C. curves it seemed, for a moment, as though the problem were becoming once more an open one. Is it that the earlier findings were after all in error?

An answer seems to be provided by the analysis of p. 55 *et seq.* It is there shown that Figs. 28*B*, 9*E*, etc., manifest beat effects within the first 5 minutes of their length owing to the coming together of many waves in the common trough at 64 seconds. It is with these same curves, in this same zone, that signs of arithmeticity are so marked. The inference is obvious. The two kinds of waves, the apparently arithmetic cycles, and the hybrid geometric ones, must be linked in some way if they appear and disappear together.

The geometric analysis is a very complete one. It is derived from the general hypothesis, which in its turn is based upon the curves of the research as a whole, not merely upon these few examples now held in question. With it, we have shown why such manifestations may come in the 64-second zone, and so on. So far, it has not been possible to conceive an explanation in terms of arithmetic waves that is in any way as complete. That of composite geometric cycles holds the field.

A problem that has continually troubled us from the beginning of the research, making us at times uncertain of our most fundamental conceptions, thus seems to be resolved. Taking into account the evidence of part and whole-duration smoothings, and of composite geometric cycles within the 64-second zone, it can be ruled that apparent arithmeticity is here probably due to a misreading of the curves.

4. CONCLUSIONS

There are signs of periods of approximately 15, 20, 30 or 60 seconds in some of the curves, such manifestations usually occurring within the first 5 minutes of work. Cases fall into two groups, the first of curves based upon many records, where the manifestations are weak, and usually of a slight crest at the ends of minutes, the other of curves based upon but few records, but in which the signs of arithmeticity may be quite strong. It is suggested that the appearance of arithmeticity in this second group is illusory, due to the coming of composite or distorted geometric cycles near the common trough point at the 64th second. Whether this completely explains the first group is not certain. A weak imposed (arithmetic) rhythm due to the use of constant time intervals is not beyond the bounds of possibility, and on summation, it could well become marked enough to be apparent, even if not noticeable in the contributory curves.

E. SMOOTHING EFFECTS; END VARIABILITY

1. The problem.
2. Data and findings.
    - (*a*) Initial damping: end variability.
    - (*b*) Geometric smoothings.
3. Conclusions.

1. THE PROBLEM

It has long been known that work-curve amplitudes increase with time. Now if the curve is essentially geometric, some of that increase in end variability must be illusory, and due merely to the use of arithmetic time intervals during experimentation. The problems of this present section have therefore to do with the estimation of that artifactual element of increase, together with certain cognate problems of smoothing.

2. DATA AND FINDINGS

(*a*) *Initial damping: end variability.*

Fig. 30A represents a geometric wave of ratio 2·00 plotted as with constant amplitudes, and against the log of time (units assumed to be of seconds). Curves B and D show that same curve expressed as average rates of output per 5 and 50 seconds respectively. In both cases, amplitudes are heavily damped to begin with, breaking free later. They can never become absolutely free as long as there are time intervals at all, but for practical purposes damping has ceased by about 50–80 or 500–800 seconds respectively.

Part of the curve of Southwark Addition (Fig. 6A) is given as

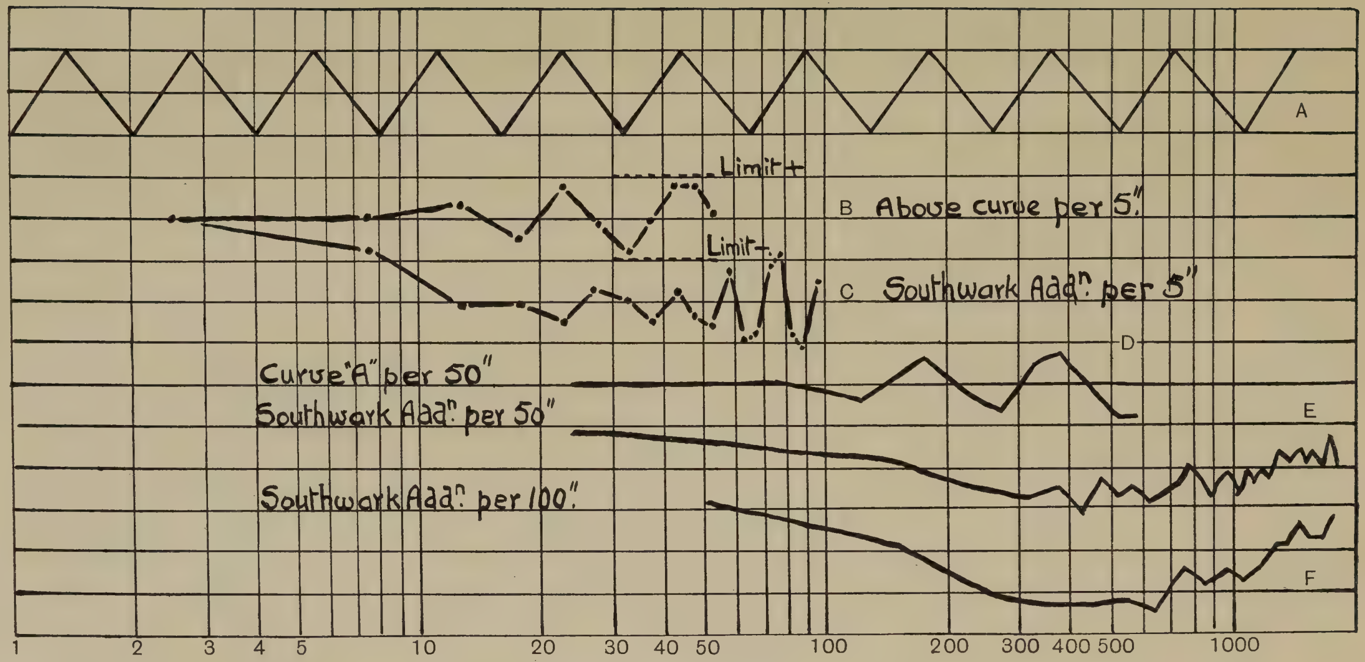

Fig. 30. Initial damping.

curve C, the original 5-second intervals being used. There is in it an appearance of initial damping roughly corresponding with that of the theoretical 5-second curve, and it too begins to be free by about 50–80 seconds.

Curve E is of the same Southwark material, this time per 50 seconds. There is obviously an increase in the zone of initial damping, and agreement with the theoretical curve is again obvious. Finally, it is given per 100 seconds that comparison may be made with the Thorndike material of Fig. 10 where the intervals, although irregular, are of this order of size. The said Thorndike curves obviously conform. (Incidentally, there is thus quite a neat demonstration of the fact that they are essentially of the same nature or kind as our own curves, despite the difference in the intervals, etc.)

Reference may also be made to the curves of Chapman and Phillips (p. 4). Both claimed to get smoothness with intervals of 120 seconds. Chapman's curve is actually smooth. It is, however, but 10 minutes long, and if there is in it a ratio of the order of, say, 2·00, it should, by this present argument, be smooth for the whole of its length. Phillips' curve, on the other hand, is twice as long. It might well therefore be expected to overrun the zone of initial smoothing, and surely enough it does begin to oscillate towards the end, although it is smooth enough to begin with.

Having shown what happens with a wave of ratio 2·00, we may infer what would happen with other waves. One of ratio $1{\cdot}94^2$ should be practically free by the 30th second. On the other hand, waves of log duration 0·0160 or 0·0016 should only begin to work loose by, say, 1000 and 10,000 seconds respectively. That is, if the intervals are of 5 seconds. If they are longer, such small waves might be imprisoned indefinitely.

There is here, of course, an argument that might be of use were we seeking the smallest possible wave. Taking an experimental interval of the smallest possible size, tests might be continued to see if increase in variability does at last tend to come to a standstill. The problem is, however, involved. To begin with, we do not know whether geometric waves themselves remain of constant amplitude. For purposes of Fig. 30 it was assumed that they do.

(*b*) *Geometric smoothings.*

Initial damping and end variability are complementary conceptions. If the underlying or true output curve has waves of constant amplitude (as indicated in Fig. 30*A*) and if the observed end variability is merely due to averaging over arithmetic intervals, then amplitudes should tend to revert to constancy when the curve is averaged afresh over geometric intervals*. Only long curves are of use in putting this notion to the test, and the longest of the research (those of Series IV) were therefore smoothed individually by half durations of ratio 1·94 (Fig. 31)†.

The individual curves are statistically weak. Chance errors are to be expected, especially within the first 30 seconds where, as usual, the original 5-second values have been plotted. At the other end of the curves, such errors will naturally tend to be smoothed out. Allowing further for the fact that the first two or three points should show strong initial damping, there appear three zones to which attention may be directed. Damped to begin with (say for 15 seconds), the curves should become variable for a time (up to say 100 seconds), thereafter steadying down to a state of more or less uniform movement.

This seems to happen. Curve *C* is a little jumpy within the first 15 seconds, but, on the whole, the curves as a group conform well enough to expectations. The exceptional curve is based upon a single record. Beginning without previous practice (although as a graduate in psychology, she was not inexperienced), the subject returned 11, 4, 6, 5, 6, sums in the first few successive intervals. There is a relatively big drop from 11 to 4, a drop that is responsible for the curve's erratic appearance. On the other hand, it is only necessary to quote the figures to show how weak they are. There are here no stable averages, based upon hundreds of records, but single digit figures in which one tiny sum more or less makes all the difference. Again, beginning without practice on the great adventure of an hour's special work, she might well have been unsettled during the first few moments. (She may even have robbed the second interval by putting a sum too many in the first one.) Yet we have used the figures of those first few seconds as though they are comparable with averages calculated over hundreds of seconds nearer the end of the experiment‡.

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* It is not a case of giving back what is lost but of introducing a complementary degree of smoothing at the end of the curve. Rectification is thus approximate.

† The exact size of the ratio is immaterial provided it is not extreme. The given intervals were chosen because a few of the curves had already been evaluated.

‡ The last three points in this curve are of average output rates over 460, 650 and 900 seconds respectively. Knowing the usual effects of averaging over large numbers of variables, it is a noteworthy fact that the end-points can even be plotted on the same scale as those near the beginning.

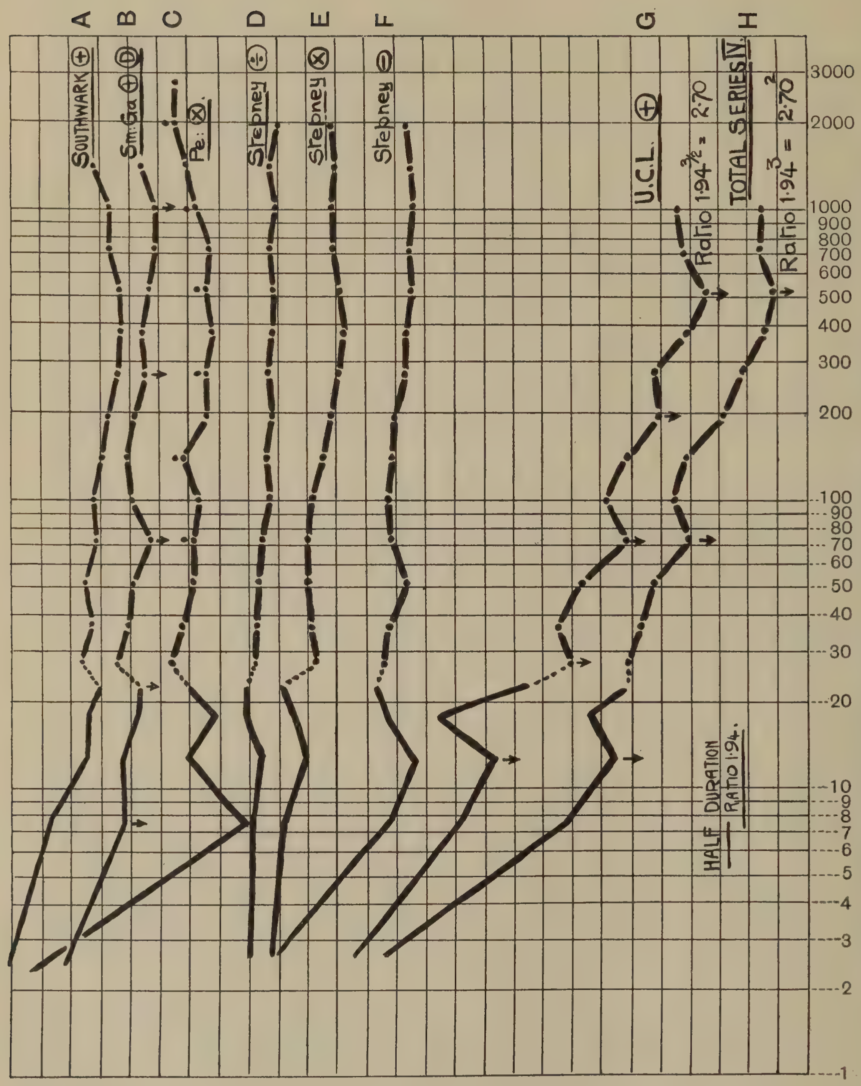

Fig. 31. Removal of end variability. Curves of Series IV, smoothed by half durations of ratio 1·94, intervals as in Figs. 9 *C* and 22.

Turning to the total curve of the series, stability is seen to have come into the initial values. Small errors from curve to curve have cancelled one another out, and points near the beginning of the curve are now definitely comparable in steadiness with the more heavily averaged ones of the end.

The rule for damping over chance-determined variables is that it is proportional to $\sqrt{N}$ (N here being the number of time intervals concerned). It is not altogether easy to deal with the curves, but the matter may be illustrated with curve C. The jump there from the second to the third point in the curve is of 5 seconds in time, and of 0·4 sums per second in output. Similarly, the jump from the last but two to the last but one in the curve is of 560 seconds, and 0·064 sums per second. Amplitude (or output) differences are roughly as 6 : 1. The roots of the times are as 10 : 1. So that, taking the worst jump in the weakest curve, corresponding movements of amplitude at the end of the curve are still almost twice as great as they should be (on the average) in chance-determined material. That being so, the average result for the group of curves as a whole is clearly well above the chance level.

It can always be argued that a naturally occurring increase in variability might be of such an extent as to be just counteracted by geometric smoothing. But if the curves oscillate regularly after smoothing that objection loses force. For no kind of chance smoothing could give consistent geometric loopings stretching from the lightly averaged beginning right down to the heavily averaged end. The total curve for the series moves perfectly by loops of ratio $1{\cdot}94^3$. There is very little of chance averaging about such a result. The weaker curves can also be brought into account. B, C and G move consistently from at least the 30th second onwards. The others do not, but they are not necessarily of this ratio.

On the whole, therefore, end variability has been eliminated without at the same time destroying any apparent geometric rhythms. In fact, they have been accentuated in the process. Incidentally, there seems to be no tendency for such geometric waves either to lose or gain in amplitude with time. Within the length of curve studied, they seem to be constant in this respect.

4. CONCLUSIONS

If a geometric wave form of constant amplitude is averaged over arithmetic intervals, it develops initial damping to much the same extent as a normal work curve of the same apparent ratio.

When work curves are smoothed over geometric intervals, the increase in end variability so characteristic of them in their ordinary form is completely absorbed.

On the whole, it is inferred that initial damping, and increase in end variability in work curves is due mainly, if not entirely, to the fact that essentially geometric material has been averaged over arithmetic time intervals during experimentation.

Since the observed increase in variability is just absorbed on geometric smoothing, it is inferred that the underlying geometric waves are nearly enough of constant amplitude throughout the length of curve tested.

F. GENERAL CONSIDERATIONS

1. Possible light on other problems.
    - (*a*) The Weber-Fechner laws.
    - (*b*) Fluctuation at the limen.
    - (*c*) Fatigue measures.
2. Possible mechanisms: conclusion.

1. POSSIBLE LIGHT ON OTHER PROBLEMS

(*a*) *The Weber-Fechner laws.*

The fact that waves in the work curve appear to be geometric irresistibly brings the Weber-Fechner laws to mind. According to Weber, the just noticeable increase in sensation dS is proportional to the corresponding increase in intensity of the stimulus dR, expressed as a fraction of the total intensity R. In other words, dS remains constant as long as dR/R is constant. Fechner expressed this by saying that $S = c \log R$. To him, this constituted a fundamental law of the transition from mind to matter.

As so stated, the laws only apply in the realm of sensory perception. There have been attempts to show that they hold also for discrimination of time intervals. It would seem that just noticeable increases in short durations may conform, but not those from tests in "equal appearing intervals."

There is obviously something of geometricity in the time measures of everyday life. Janet has said that the passing of a year to a lad of ten is relatively as long as four years to a man of forty. The same kind of thing may happen with shorter intervals. Setting out on an hour's train journey, one may look up with surprise on passing the first station out, realising that although it seems an age, there has passed only the first stage of the run. Later, when the trip is nearly over, one begins to pack up when still several stations from the end, time now seeming to fly by. If this is to be interpreted in terms of psychological pulses, then obviously it is due to the fact that there were many geometric pulses during the journey to the first station, perhaps only a part of one during the interval covered by several stations at the end.

In supposing that things might happen in this way because of geometric pulses in attention, account has to be taken of the psychological starting point. If, subjectively, the journey has begun some time before the actual moment of starting, then intervals between stations will appear more equal as measured in ordinary (or arithmetic) time intervals*. Any such displacement of the zero point must result in measures that lie between equality on the arithmetic and logarithmic scales respectively.

Coming to experimental work, taps may be made at times T_0 and T_1, the subject being required to make a third tap at a time T_2 such that the two intervals seem to him to be equal. If the process is determined by geometric waves, then the second interval ought to be more or less equal to the first on the logarithmic scale. Or so at first sight one might be inclined to think. In this particular case, things are complicated by the fact that the log of zero is minus infinity. If T_0 is a true zero on the arithmetic scale of time, the second interval (T_1–T_2) could not possibly be equal to the first on the log scale.

Another difficulty, a very interesting one, is that new waves may be set up at the point of change. This has been shown to happen when the subject changes from one task to another in sandwiched work. It may happen here also, the times T_0 and T_1 seeing the origination of two different sets of waves. If so, then there will, as it were, be one little "work curve" between T_0 and T_1, another beginning at the latter point to terminate at T_2. Clearly, if the two sets of waves happen to be of corresponding L.C.M. cycles or periods on the log scale, they may be judged to be of equal length in seconds of time, even although the subject was working in logarithmic units*.

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* The logs of 10 and 20 seconds differ more than do the logs of $(X+10)$ and $(X+20)$ seconds. As X increases in size, so relative differences decrease until finally, differences on the scales of logs and antilogs become practically proportional to one another.

The notion of time estimates in terms of little nests of waves leads to another problem. Suppose there is a sign or clue attaching to a trough, making it a suitable place at which to end a stimulus interval. It would follow that if we test a subject with intervals of varying length, we must find that he responds to some more accurately than to others, the ones best judged being of intervals terminating at one of the frequently occurring troughs. Moreover, of course, if the subject tends to respond at troughs, he may answer with reference to the nearest trough, even if the stimulus in question happens to fall just short of or over that point. In other words, he may over-estimate times short of it, under-estimating times over it. There will be variations both in respect of constant and variable errors.

There is here possibly a contact with work on the indifference point, and associated points of maximal accuracy in time judgments†. Estimates are said to occur most accurately with an interval of just over 0·70 second, or multiples thereof.

The problem may be illustrated by enquiring what troughs should occur in this neighbourhood. Taking a ratio 1·94 sequence as likely to be most important‡, values are obtained as follows:

0·61	1·19	2·38	5·53	8·40	seconds
77	78	79	80	81	indices

At 0·61 second, waves of indices contained in 77 should be meeting in a common trough. Many of the major ratios are of this group (see Table IV) and the resulting depression should be well marked. The sequence of times§, taken as a whole, looks very much like some of

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[*] A number of interesting speculations derive from this fact that geometric waves can be generated afresh at points of change. What constitutes such a point? One could conceive, for example, that even an elementary thing like one sum in Kraepelin sheet will see the generation of fresh pulsations. In such a case, every ordinary work curve would, as it were, be a complex "Full" curve, containing a whole series of "Extracted" ones, each starting from one of the work units. Another problem is of the extinction of waves. What happens to them when work nominally comes to an end? (cf. p. 113).

[†] Times of less than 0·70 second tend to be over-estimated, times greater than it being under-estimated.

[‡] The deepest point in any given zone is not necessarily of the ratio 1·94 sequence. The given figures are quoted as illustrative values. See Appendix B.

[§] The figures are by extrapolation. Ratio 1·94 has probably been measured with sufficient accuracy for ordinary work-curve ranges, but it does not follow that it can be trusted as far away from the centre of the curve. A slight error in the ratio, and the position of the point of index 77 will be considerably changed.

We have, of course, no justification for assuming the Indifference Point to be associated those published (cf. p. 6). It has not, however, been possible to take the problem further. If there proves to be anything in the notion, the Indifference Point may have some connection with the trough of index 77 (or nearby point in the curve), while points of maximal accuracy in time judgments will follow it in geometric succession, and not (as suggested by other authorities) in an arithmetic series.

(*b*) *Fluctuation at the limen.*

Guilford (6) claims that fluctuations are "nothing but a matter of the limen." His subjects sat for 3-minute tests with lights of minimal intensity. With a faint light, there were, of course, but occasional spells of visibility. The mean period of visibility was thus small, that of invisibility large. Proceeding up the scale of intensities, the percentage of visibility was found to vary according to a curve of γ-ϕ form. Turning next to complete periods, the work of Wiersma was confirmed (p. 6), finding that the "total period" is least at the limen, increasing as that central value is left in either direction.

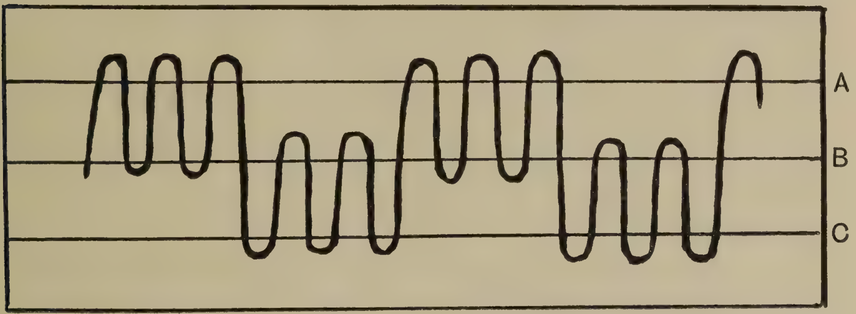

Fig. 32. Of waves at the limen.

Guilford says that the findings are hard to explain in terms of fluctuations of attention, although they might be explicable as the result of the interplay of chance factors. In that, one cannot agree. Apart from the question of attention as such, it is quite easy to imagine the findings to derive from oscillatory material.

The accompanying diagram (Fig. 32) is of a simple wave form. Let the line *B* represent the level of liminal stimulus intensity. From the way in which that line cuts the waves, it is clear that all three of his "periods" will be small at that level. As we go outwards, to levels *A*

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either with a crest or a trough in the curve. The significance of the already published results is in their constancy. It sounds as if they may be associated in some way with curves of constant wave-length and phase, hence the above tentative reasonings.

or *C*, it is as clear that the coming of long spells of visibility or invisibility will tend to disturb one of the contributory periods more than the other, the net result being to lengthen the "total period." A similar diagram could even be used to explain the γ-ϕ relationship. By making appropriate alterations (the present curve is a little formal) a wave could be constructed to give an approximation to the results mentioned by him.

Incidentally, we thus show how with one and the same wave form (i.e. with no change in the real periods) there may yet seem to be changes in the "total period" when the method used by Guilford (and others) is employed.

To say that fluctuations are of the limen is rather curiously to invert the natural order of things. One would rather say that fluctuations may attend the limen. It might perhaps be said that the differential limen is of fluctuations. But certainly not the reverse. Assuming that the limen is of fluctuations in this sense, one or two riders may be added. Results obtained from experiments such as those just described should be plotted on paper graduated logarithmically both ways. For by the Weber-Fechner laws stimulus intensities vary logarithmically, while by our own findings time measures follow suit.

If so plotted, liminal curves for all possible intensities (within the range for which Weber's law holds) should appear as a series of parallel curves, curves in which both amplitudes and periods are constant. Moreover, since the basal curve is roughly hyperbolic, it should tend to appear as a straight line, the fatigue decrement thus being measurable in terms of a simple linear expression*.

(*c*) *Fatigue measures.*

Quite a few writers have assumed increase in end variability to be of fatigue. By our own conception of the matter, as elaborated in Section E above, the manifestation is due mainly if not entirely to the fact that geometric material has been assessed in terms of arithmetic intervals.

In making that assertion account was taken only of what happens in curves of continuous output. There is nothing in the findings to throw light upon increase in variability as between, say, morning and afternoon work. If, as is quite possible, waves generated in the morning can go on into the afternoon, cutting across periods of apparent "rest," and so on, then afternoon work would of necessity seem to be more variable. The problem is an interesting one, and awaits research in the matter of the extinction of waves, together, of course, with an enquiry into what happens during rest or sleep.

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* The conception here is of liminal curves alone. How far ordinary work curves could be plotted in this way is another matter. Flugel (15) has shown that as the level of successive work curves is raised by practice, so amplitudes rise as well, but not in proportion. The problem here of course is that of making measures of difficulty or of facility that shall be comparable with those of intensity on the sensory side.

Dodge's law with regard to switching from fatigued to unfatigued mechanisms has already been mentioned. His ideas are perhaps illustrated by the introspections of subject Pe. (Series IV). After working for an hour at Multiplication, she averred that her curve could not possibly be regular, since she had varied in her methods of working. Sometimes she had "multiplied" the numbers, sometimes she had "just known the answer," and so on. As it happens, her curve was as regular as any. The observed switchings from one way of "doing" the task to another were here possibly the introspectible concomitants of a definite switching from one mechanism to another. But there is no proof of such a notion. With rivalry experiments we are on somewhat safer ground, since, by definition, times during which one aspect of an ambiguous figure is dominant have been taken to be positive, and *vice versa*. Again, however, there are difficulties. We cannot say that there were only two mechanisms at work, or even, if so, why the corresponding waves should interlock, as they seem to do, to form one consistent curve*.

It has been reported by various writers that subjects can voluntarily hold one aspect of an ambiguous drawing longer than another. Or that by adding detail to the drawing (converting, say, a skeleton truncated pyramid into a picture of a corridor with pictures on the walls, etc.), one aspect may be made to dominate awareness so completely as to exclude the other. Evidence of this kind should not be interpreted to mean that periodicities as such can be controlled. By Dodge's law, one would assume it to be in part a matter of decrements. It can also be read as having to do with a raising or lower of the limen. And from the discussion of Guilford's work above, it is clear that there need be no change in the underlying periodicities, even if, with our crude measures, the apparent periods differ from test to test. As far as that goes, of course, the whole trend of this present research is against such a view. The system we have built up would break down at once were periodicities variable or controllable in this sense.

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* Output curves of the kind dealt with in this work are not of the total amount of energy expended, but merely of output in the performance being measured, a very different thing. It is conceivable, for example, that the subject's energy was being expended at a constant rate throughout any of our tests, but that the amount given to the task formally in hand varied from moment to moment, any not so utilised being taken up by the subject's private thoughts about the task, his day dreamings, and so on. One cannot say. All that is certain is that output in the allotted tasks varied according to the curves produced.

It was suggested when discussing the Extracted results (p. 75) that there may be a decrement beginning at the point of change. If this decrement is associated with the individual waves, the question arises whether the decrement (and fatigue) can be either specific or general save in the sense in which we have said that waves are of general utility although possibly specific to as yet unnamed factors or mechanisms. Phillips (14) says that objective fatigue is general in some ways, specific in others*. It is possible that our suggestion of a decrement beginning with the point of change in a sandwiched task is on all fours with his findings. The problem calls for further research.

2. POSSIBLE MECHANISMS: CONCLUSION

It is difficult to imagine a mechanism, physiological or psychological, covering the findings as a whole. The cycles usually mentioned when discussing fluctuations, namely those of the pulse, of respiration, and so on, are of arithmetic periods. It seems clear that none of these can be concerned.

An early conception of our own was that waves are arithmetic at bottom, but that as deeper and deeper sources of energy are tapped, so more and more waves appear, each of longer wave-length than the one before. If early waves die as they are replaced, then there might be an appearance of increasing periodicity in the resulting curve. The difficulty is that there is no apparent reason why waves so generated should seem to be rigidly geometric. An alternative notion is of a slowing forced upon the waves by mechanisms such as those of fatigue, or as in the last conception, of drainage. Again, however, the difficulty is to show why it should of necessity be a geometric slowing.

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* Phillips' own figures and findings are curiously involved. They may be interpreted by saying that fatigue is engendered specifically, but transferred (in part) generally. (See Spearman (36), chap. XVIII.)

Incidentally, there may be a flaw in Phillips' method. His subjects worked continuously for 90 minutes at one or other of four fatiguing tasks, the work in each case being preceded and followed by tests in eight different kinds of work. He then compared output in each of these kinds of work before and after the fatiguing task to see whether there is transfer of fatigue, and so on. Before correlating, however, he made a correction to each subject's work to allow for the amount of practice that would have accrued had the given tasks not been interrupted by the fatiguing work. These corrections seem to have been of constant amount (per subject per task) whatever the nature of the fatiguing work. In correlating, he would thus get a constant value on either side in some of his series. And it is important to note that where this could happen he gets a positive correlation, where it could not he averages zero correlation. As no figures are quoted, it is not possible to check the matter.

His general conclusions may, of course, be sound, despite the apparent flaw. Our own findings, for example, run with his to some extent, although our tentative interpretation (as given above in terms of "general utility" waves), is different.

There are bodily mechanisms such that they only act when fully charged*. Remaining quiescent until a certain pressure has been attained, they go over into action, returning thereafter to a fresh period of re-charging. If the cycle of charge and discharge can be assumed to begin at zero time, there might here be a mechanism that would produce curves of fixed phase constants, while if the periods slow down with fatigue, etc., there might be an explanation of increasing periodicity, although again there is no necessary condition ensuring rigid geometricity.

As another kind of notion, there may be alternating mechanisms such that we go from one to another as exhaustion sets in. If these work together in pairs (or groups), they could perhaps act and react in geometric time sequences.

On the whole, however, considerations of this kind seem inadequate. The extraordinary system of relationships between the ratios, the existence of a real or virtual unit of periodicity, all testify to the presence of mechanisms of hairspring fineness, mechanisms of a different kind altogether from those of simple cell mechanics, of drainage, or the like.

It is as though we set out to measure more or less crudely the percentages of malic acid in a number of apples taken individually, finishing up with an analysis of malic acid itself, a thing independent of the apples from which it is drawn. In similar fashion, we seem to have derived something from our various subjects that is dependent not upon their individual characteristics, but upon factors common to all humanity.

That "something" eludes us. On the one hand, it may after all be of the environment. If that is so, it must be a very fundamental something. It appeared in the cold and draughty French house in which subjects Sm. and Ga. worked, great-coated, and huddled close to a smoky oil lamp with a broken chimney. It appeared whether they were dotting, adding, or viewing an ambiguous drawing. It appeared in the quietness of the laboratory, and in the crowded class rooms of an East End school, where street noises and occasional bursts of song from the infants below came rising to the top floors. It appeared in the curves borrowed from Thorndike. If there is such a fundamental environmental factor, we have demonstrated the way in which its manifestations can be read and systematised.

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* See Spearman, *Abilities of Man*, p. 327.

On the other hand, the waves may be of the psychophysical mechanism as such. If so, then returning to the analogy of the apples, we must inevitably look for a source in such things as complex molecular changes or the like, things the laws of which could be of universal application, and true for all humanity.

In such a case, the work curve has, in effect, been shown to be analogous, if only remotely, with the spectrum of physics. In place of Frauenhofer lines, we have our trough sequences. What they mean, we cannot say. They may ultimately be associated with given psychophysical functions, enabling the curve to be used for analytical purposes, just as the astronomer uses the spectrum. One cannot see as far ahead. Or again, just as physics has her quantum theory, given wave-lengths being associated with given energy amounts, so the finding of our unit of wave-length may foreshadow the emergence of a quantum theory in things psychophysical. Again, one cannot say.

We have not been able to explain the manifestations. Fortunately it is not necessary to do so. It is sufficient to have described and systematised them. It has been well said that Nature is tolerant of man's hypotheses, condescending to work by them as far as she is able, even although the mental pictures behind them may be inadequate. The system may not be as we have pictured it. But, on the other hand, it can be said to work. We have, as it were, constructed a mill. Steam being let in, the wheels have turned, however clumsily, grinding out a table of trough sequences apparently true for all the curves met with. Whether the general hypothesis is adequately expressed or not, whether the waves are strictly geometric or not, whether we can explain the facts or not, the wheels have turned. There is a pragmatic sanction.

APPENDIX A

NOTES ON CURVES OF SERIES I TO IV

Ratio 2·00 Group.

Sm. Addition.	14	records	Fig. 7.
Sm. Dotting.	4	,,	8.
Sm. Illusion.	1	,,	9.
Thorndike material	2	,,	10.

The first two cases are of the same general type as Example 1 of the text (Fig. 2). Smoothings are by one-half and whole durations. One-third duration smoothings (not shown) gave results similar to those of Figs. 2 and 5, with signs of ratio 2·52 in the curve of Sm. Dotting corresponding to those in the curve of Py. (Fig. 3). Double humping occurs in the half-duration curves, the pairing of crests being as for Fig. 5*C*.

The curves were said to be of ratio 2·00 because of the presence of the characteristic crest sequence (45, 90, 180, ... seconds). For comments on the curves of Fig. 7 in this respect see p. 51 *et seq.*, where it is suggested that this example has troughs characteristic of ratio 3·02.

Fig. 9*A* is of part of the original record from this experiment (time scale magnified five times as compared with remaining curves). Times during which the first appearing aspect of the ambiguous cube was present were taken to be positive and *vice versa*. Curve *B* is of spells. Curves *C* to *F* are of various smoothings. There are signs of the 11, 22, 45, 90, ... crest sequence in *C*, and of the troughs of ratio 1·94 in *D*. With curve *E* an attempt was made to trace a resemblance to Fig. 6*A* by means of one-third durations of ratio 2·09. Instead of a threefold looping, however, there came a fourfold one, the approximate ratio thus being $2{\cdot}09^{\frac{4}{3}} = 2{\cdot}67$. For further comments on this ratio see p. 62.

The Thorndike curves are from figures given in the *Psychological Review* (19, p. 167). Results are there quoted in terms of times taken to do unit work. For our purposes, reciprocals of given times were plotted against their centres on the time scale (Fig. 10, curves *A* and *B*). The intervals being coarse, the curves only begin to be at all detailed from the 1000th second onward. Comparison may be made with curves of Fig. 9 where times were accurate to the nearest half-second. Half-duration smoothings of ratio 2·00 (curves *C* and *D*) are apparently in opposition. Calculations are here on the assumption that there are crests in curve *B* at approximately 750, 1500 and 3000 seconds, running serially with those at 180, 360, 720, ... seconds in, say,

Fig. 5. There is also association with ratio 2·40, or a power thereof, points of approach and recession in curves *A* and *B* being marked in terms of the ratio 2·40 sequence as established in the Hi. curve of Fig. 4. (If the trough sequence of the latter curve is calculated forward it will be found to join on with that of Fig. 10.) It is interesting to note that the Hi. curve also shows signs of ratio 2·00. For another case of association between ratio 2·40 and the ratio 2·00 group of cycles see curves *E* and *F* of Fig. 18. The latter is in opposition to the standard ratio 1·94 series (cf. Fig. 31*C*).

Ratio 3·02 Group.

Four Boys Addition.	23 records.	Fig. 11.
Sm. Dotting.	14 ,,	12.

Whole and quarter-duration smoothings are given. The latter show signs of a minor cycle of ratio $3{\cdot}02^{\frac{1}{2}} = 1{\cdot}74$. As noted on p. 62, there is apparent opposition between Figs. 11*A* and 12*A* as far as the major cycle is concerned, although waves of the minor cycles are in step. The theoretical explanation is that the curve of Sm. Dotting should be returned as more of ratios 3·46 or 3·76, thus being removed from the present family.

Ratios 2·40, 2·52 and 2·70.

Four Boys Sound.	19 records.	Fig. 13.

Only one major curve of the earlier series has a ratio of this group. For other examples see Table V.

The standard cases of the first two ratios are of Hi. and Py. in Fig. 3. Later identifications were in terms of the trough sequences there indicated, although the curve of Fig. 13 seemed to stand apart. Ratio 2·70 was added to the group when it was seen that, on one occasion, we had returned the curves of Fig. 20 as being of ratio 2·52 and on another as of 2·70. We thus empirically placed the cycles with troughs at

62, 150, 355, ...	seconds	(ratio 2·40).
67, 170, 425, ...	,,	(,, 2·52).
66, 180, 480, ...	,,	(,, 2·70).

On this basis, the curve of Four Boys Sound would be more of ratio 2·70, although its smoothings were not altered to conform. For theoretical analysis, see Table V.

Ratio 1·70 Group.

Six Boys Dotting.	69 records.	Fig. 14 (less curve *E*).
Ga. Addition.	14 ,,	,, 15.
Ga. Dotting.	8 ,,	,, 14*E*.

Curves of Fig. 14 were among the most troublesome of the research. Taking crests and troughs of Fig. 14*A*, and reducing the times at which they occur to logs, it was found (by successive differences) that the best fitting common factor lies between, say, 0·0072 and 0·0070. Taking 0·0071 as a mean value, the major and minor cycles seemed to be of 16 and 32 times that value, i.e. of 0·1136 and 0·2272, or of ratios 1·30 and 1·69 (Table II).

By trial and error, it was found that the minor waves of Figs. 14*C* and *E* become blurred if we use wave-lengths other than 0·1120 and 0·2240 (ratios 1·30 and 1·68). Smoothings of Fig. 15 were made with the same intervals, although here, ratio 1·70 as such could have been used.

It was early recognised that curves of this group are related to those of ratio 2·00. (See, for example, crests at 22, 45, 90, ... seconds in Fig. 15*A*, or at 22½ and 90 seconds in Fig. 14*E*.) Such a relationship is to be expected, since $1{\cdot}70^5 = 1{\cdot}94^4$. The problem was attacked by smoothing all the major curves of the research in terms of one-fifth durations of ratio 1·94 (= quarter durations of ratio 1·70). Three such curves are given as *D*, *E* and *F* of Fig. 6. From this work, it became apparent that the curves of Fig. 14 stand apart, it being more usual for curves to be of the type of Figs. 6 and 15. (There is a phase difference of a quarter duration between Figs. 14*C* and 15*C*.)

The 'shapes' of Figs. 24 and 25, although used in the text to illustrate other problems, were really a stage in the attack in this present question of wave-lengths of the order of 0·1100 to 0·1152.

Later, of course, came the theoretical analysis of p. 55 *et seq.*, in which it was shown that cycles of this order of size might be of composite origin.

Ratio 1·50 Group.

Ll. W. J. Sound.	4 records.	Fig. 16.
Ga. Dotting.	14 „	„ 17.

The Ga. curve includes the eight records of Fig. 14*E*. Smoothings of half durations of ratio 1·50 loop consistently from the 25th second onwards. Ratio $1{\cdot}50^3$ seems to be present in Fig. 17, but with a different trough sequence from that in Fig. 6*B*. See theoretical discussion, p. 51 *et seq.* Whole-duration smoothings do not clear the curves. Treble and quadruple intervals respectively were used.

For other examples see Table VIB.

Curves of Series IV.

Sm., Ga., Addition and Dotting.	4 records.	25 minutes.	Figs. 5, 6, 31.
Southwark. Addition.	6 ,,	30 ,,	,, 5, 6, 31.
Stepney. Division.	2 ,,	40 ,,	,, 18, 31.
,, Subtraction.	2 ,,	40 ,,	,, 31.
,, Multiplication.	2 ,,	40 ,,	,, 18, 31.
Pe. Multiplication.	1 ,,	60 ,,	,, 18, 31.
U.C.L. Addition.	8 ,,	22 ,,	,, 31.
Total Series IV.	25 ,,	22 ,,	,, 31, 21.

The first two cases are given in the text as Example 2. The others are used to illustrate various problems.

Curve *A*, Fig. 18, is plotted on log paper in full detail. It illustrates the tendency to end variability, probably in somewhat extreme form.

For various reasons it was smoothed by units of 0·0360 (curve *B*). There is a tendency for loopings of 3, 6, 9 or 12 of those units to appear.

A twelvefold looping would be of ratio 2·70. Assuming that ratio, quarter durations (log intervals of 0·1080) were tried. The resulting curve (*C*) loops regularly by threes and fours alternately. Ratio $2{\cdot}70^{3}$ seemed to be indicated. Quarter durations of this ratio (curve *D*) move consistently in a faint up-and-down rhythm.

Curves *E* and *F* of this plate are in opposition for the first 130 seconds of their length. See discussion of Table IX.

APPENDIX B

TABLE OF THEORETICAL TROUGH SEQUENCES. A SCALE OF LOGARITHMIC TIME

Calculated trough sequences are given in Table X for wave-lengths within the range of those usually met with.

They are plotted in Fig. 33. Horizontal measures are of seconds of time, vertical ones of wave-lengths. To save space, only even-numbered wave-lengths are given. If it is desired to find troughs of wave-length 200, a line should be drawn across at a height of 200 units, noting where it meets plotted points (points must be sought, not intersections with any of the lines joining points). And so for any even-numbered wave-length. (It would be possible to get odd-numbered values by taking account of the systematisation of the points at the desired level.)

Near the left-hand side of the diagram, trough points fall into curves of cocked hat shape. Towards the right-hand side, they are more scattered, but faint lines joining them show that the same general arrangement holds.

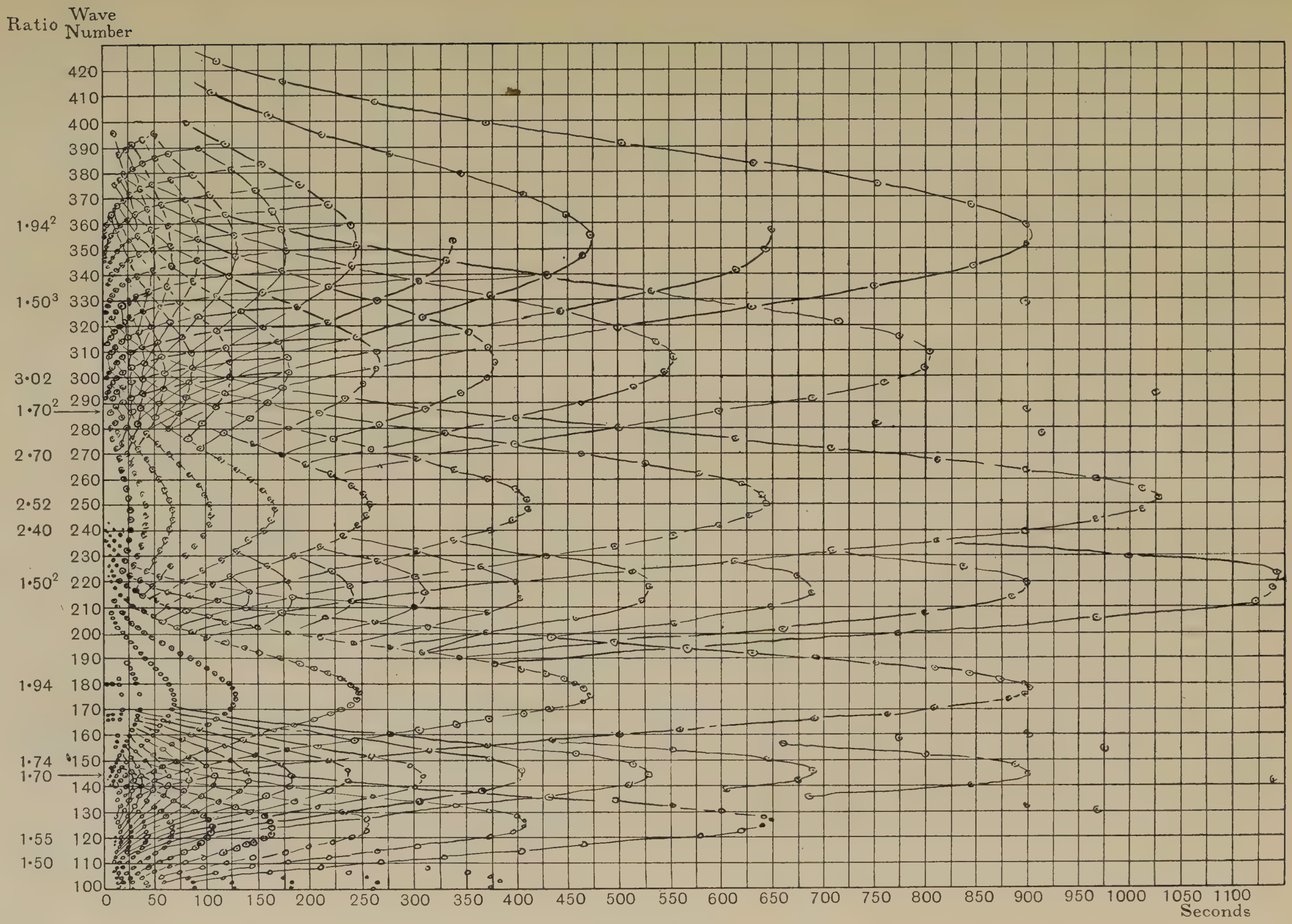

Fig. 33. Theoretical distribution of troughs.

Troughs clustering in the 180-unit zone plot out a very cleanly cut "nest" of curves. On the other hand, those in the 140–150 unit band (covering ratio 1·70) have troughs that seem to be hopelessly confused.

Troughs scatter themselves differently at the various levels. In the 60–66 band, they plot out curves in which each single wave is represented. In the 170–190 band, waves have to be taken alternately, even-numbered lengths plotting out one nest of curves, odd-numbered ones another. And so on.

The diagram is of no immediate use in analysis, but is given to illustrate the complexity of the trough system in a work curve containing all the possible waves in equal weight.

The problem of nomenclature is of interest. If all waves come to a common trough at the point T_0, then for all practical purposes that point may be taken to be zero on the scale of log time units. It was in this sense that, using wave-length 180 as a unit, the 64th and 900th seconds of ordinary time were taken to be 84 and 88 standard cycles from T_0. Save for the fact that the figures become somewhat large, wave-length 1 might more logically have been taken, the two times mentioned then being given as 15,120 and 15,840 log units from zero respectively.

It was pointed out in the text (p. 45 *et seq.*), that all cycles of indices contained in 84 must come to a trough at a point 84 standard cycles from zero. Using wave-length 1 as a unit, and stating the same argument more generally, all waves whose lengths are factors of 15,120 must come to a common trough at that point. In other words, we have merely to scale the time line in terms of the given unit of wave-length, the number of possible waves coming to a trough at any point then being given by the number of factors in the time value concerned.

If we assume that actual amplitudes may be ignored, it follows that the depth of what may be called the theoretical curve at given points will be roughly proportional to the number of waves there coming to a trough. In this way it should be possible to arrive at an approximation to the outline of the said theoretical curve. How far such a result could be correlated with actual work curves is another matter. To begin with, we do not know how large the hypothetical pool of elementary waves may be, how many of the possible waves within that range may be present, or the relative frequency with which they occur. The problem is, however, worthy of investigation.

Table X

Wave-length	Ratios (powers) 1st	2nd	3rd	Troughs (seconds)
100	1·44	__2·09__	_3·02_	__41__* 59 __85__ _123_* __178__ 257 __372__* 537 __777__ _1123_*
102	1·46	__2·12__	_3·09_	_40_* __58__ 85 __123__* 179 __261__ _380_* __553__ 806 __1173__*
104	1·47	__2·15__	_3·16_	__37__* 55 __80__ _118_* __173__ 253 __372__* 545 __800__ _1173_*
106	1·48	__2·18__	_3·23_	_33_* __49__ 73 __108__* 159 __235__ _348_* __514__ 760 __1123__*
108	1·49	__2·22__	_3·30_	43 __63½__ _94_* __141__ 209 __312__* 464 __690__ _1033_*
110	1·50	__2·25__	_3·37_	53 __79__* 119 __178__ _267_* __400__ 600 __900__*
112	1·51	__2·28__	_3·45_	42 __63½__* 96 __145__ _219_* __330__ 499 __754__*
114	1·52	__2·32__	_3·53_	32 __49__* 74 __113__ _172_* __261__ 397 __604__* 920
116	1·53	__2·35__	_3·60_	_36_ __55__ 84 __129__* 197 __302__ _464_* __711__ 1090
118	1·55	__2·39__	_3·68_	39 __60__ _91_* __143__ 220 __340__* 526 __812__
120	1·56	__2·42__	_3·77_	41 __63½__* 99 __154__ _239_* __372__ 578 __900__*
122	1·57	__2·46__	_3·85_	_42_* __65__ 102 __160__* 252 __394__ _618_* __969__
124	1·58	__2·49__	_3·94_	41 __65__ _103_* __163__ 257 __406__* 641 __1013__
126	1·59	__2·53__	_4·03_	__25__ 40 __63½__* 101 __160__ _255_* __406__ 646 __1028__*
128	1·60	__2·57__	_4·12_	_37_* __60__ 96 __154__* 246 __394__ _632_* __1013__
130	1·61	__2·61__	_4·21_	34 __55__ _88_* __143__ 230 __372__* 600 __967__
132	1·63	__2·64__	_4·30_	30 __49__* 79 __129__ _209_* __340__ 553 __900__*
134	1·64	__2·68__	_4·45_	_26_* __42__ 69 __113__* 185 __302__ _495_* __811__
135		__2·70__	_4·50_	_39_* __63½__ 104 __172__* 282 __464__ _763_*
136	1·65	__2·72__	_4·55_	__35__ _58_* __96__ 158 __261__* 431 __711__ _1173_*
137		__2·75__	_4·60_	32 __53__ _87_* __145__ 240 __397__* 658 __1090__
138	1·66	__2·77__	_4·65_	__29__* 48 __79__ _132_* __219__ 364 __604__* 1005
139		__2·79__	_4·70_	26 __43__* 71 __119__ _198_* __330__ 551 __920__*
140	1·68	__2·81__		38 __63½__* 106 __178__ _298_* __499__ 836
141		__2·83__		__33__ 56 __94__* 159 __267__ _449_* __754__
142	1·69	__2·85__		_29_* __49__ 83 __141__* 237 __400__ _675_* __1140__
143		__2·87__		__25__ _43_* __73__ 124 __209__* 354 __600__ _1016_*
144	1·70	__2·89__		__37__ _63½_* __108__ 183 __312__* 529 __900__
145		__2·91__		32 __55__ _93_* __159__ 272 __464__* 791
146	1·71	__2·93__		__27__* 47 __80__ _138_* __235__ 403 __690__* 1182
148	1·73	__2·98__		34 __58__* 100 __173__ _298_* __514__ 887
150	1·74	__3·02__		__41__ 71 __123__* 214 __372__ _646_* __1123__
152	1·75	__3·07__		__28__ _49_* __85__ 149 __261__* 457 __800__
154	1·76	__3·11__		32 __57__ _101_* __178__ 314 __553__* 975
156	1·78	__3·16__		21 __37__* 66 __118__ _209_* __372__ 660 __1173__*
158	1·79	__3·20__		__24__ 42 __76__* 135 __242__ _434_* __776__
160	1·80	__3·25__		_26_* __47__ 85 __154__* 277 __499__ _900_*
162	1·82	__3·30__		__29__ _52_* __94__ 172 __310__* 566 __1028__
164	1·83	__3·35__		31 __56__ _103_* __189__ 345 __632__* 1156
166	1·84	__3·40__		__33__* 60 __111__ _205_* __377__ 695
168	1·86	__3·45__		34 __63½__* 119 __219__ _406_* __754__
170	1·87	__3·50__		__35__ 66 __123__* 230 __431__ _806_*
172	1·89	__3·55__		_36_* __67__ 127 __239__* 450 __848__
174	1·90	__3·60__		__36__ _68_* __129__ 244 __464__* 879
176	1·91	__3·66__		35 __67__ _129_* __246__ 471 __900__*
178	1·93	__3·71__		__34__* 66 __127__ _244_* __471__ 906
180	1·94	__3·77__		33 __63½__* 123 __239__ _464_* __900__
182	1·96	__3·80__		__31__ 60 __118__* 230 __450__ _880_*
184	1·97	__3·88__		_29_* __56__ 111 __219__* 431 __848__
186	1·99	__3·94__		__26__ _52_* __103__ 205 __406__* 806 __1593__
188	2·00	__4·00__		24 __47__ _94_* __189__ 377 __754__* 1502
190	2·01	__4·06__		__21__* 42 __85__ _172_* __345__ 695 __1401__*
192	2·03	__4·12__		__37__* 76 __154__ _312_* __632__ 1281
194	2·04	__4·18__		32 __66__* 135 __277__ _566_* __1156__
196	2·06	__4·24__		__28__ 57 __118__* 242 __499__ _1028_*
198	2·07	__4·30__		_23_* __49__ 101 __209__* 434 __900__

Second power troughs in heavy type.
Third power troughs *.

APPENDIX C

RUNNING MEASURES

Running average. If a curve is thought to contain a wave of 11 units period, it can be cleared of that wave as follows. Take the average of points 1 to 11, following with the averages of points 2 to 12, 3 to 13, and so on. These successive averages plotted against the centres of the intervals concerned give the curve of Running Average.

Deviations from a Running Average. If it is desired to accentuate a wave of, say, 11 units period, expressing it more or less free from minor irregularities, or from slope in the basal curve, the original figures may be expressed as deviations from corresponding points on the curve of Running Average.

Running Deviation. Proceed as for a running average, taking next the mean deviation of points 1 to 11 from their mean, the mean deviation of points 2 to 12 from their average, and so on. (This is by no means the same thing as the second curve above.) Other smoothing intervals may, of course, be taken, or other measures of deviation used. (During the research, the measure employed was the standard deviation. No curves of this type have, however, been quoted in the text. Their use is to measure the rate of increase in variability or amplitude swing.)

Running Shapes. Taking several copies of a curve, they are placed under one another, each one cycle length in advance of the one next preceding it. On summation, a composite curve is obtained, one that is in effect a series of "shapes" joined together. We thus show how shape outlines may vary from one part of a curve to another, or how end-point placings may vary. No examples are quoted in the text, results having conformed to those obtained with the ordinary kind of shape mentioned on p. 77 *et seq.*

Running Correlation. As before, taking points 1 to 11 (or other convenient grouping), correlate corresponding points in two given curves. Follow with correlations between points 2 to 12, 3 to 13, etc. Plotted against the centres of the intervals concerned, such successive correlations show whereabouts in their outlines the two given curves tend to agree or disagree.

The method was used during the research to see whether periodograms from total curves of series tend to have any wave-bands in common. By summing all the obtained curves of running correlations, it was further hoped to distinguish wave-bands in which agreement is greater than usual, etc. The results had unfortunately to be scrapped, the original periodograms proving to have been calculated from faulty data. One did not feel justified in replacing more than the periodograms themselves. Although the said running correlations are too faulty to be produced here, it may perhaps be reported that there was a tendency to greater correlation with long cycles than with short. This one would expect on the hypothesis of L.C.M. repeat periods.

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